Overview on Top and Single Top production

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I.: Basic facts about the top
II: Top-pair production
III: Single top production
IV: Outlook
Basic facts about top

The essential numbers:

Mass:

\[ m_t = 173.07 \pm 0.52 \pm 0.72 \text{ GeV} \]

Width:

\[ \Gamma = 2.0 \pm 0.5 \text{ GeV} \]

Branching fraction:

\[ \frac{\Gamma(W b)}{\Gamma(W q)} = 0.99^{+0.09}_{-0.08} \quad \text{(pre-2011)} \]

\[ = 0.90 \pm 0.04 \quad \text{(D0, 2011)} \]

\[ = 1.023^{+0.036}_{-0.034} \quad \text{(CMS, 2012)} \]

Discovered at the Tevatron in 1995

A $t \bar{t}$ event from CDF.
Implications

The top quark is special:

- The large top mass sets a hard scale.
- Lifetime shorter than characteristic hadronization time scale.

⇒ Top physics is described by perturbative QCD.

- The top mass is close to the electro-weak breaking scale.

⇒ If there is new physics associated with electro-weak symmetry breaking, top physics is a place to look for.
Theoretical aspects of the top mass

\[ \mathcal{L} = \bar{\psi} (i\not{D} - m_{\text{bare}}) \psi \]

Renormalisation:

\[ m_{\text{bare}} = Z_m m_{\text{renorm}} \]

\( Z_m \) and hence \( m_{\text{renorm}} \) depend on the renormalisation scheme!

Conventional choices:

- The \( \overline{\text{MS}} \)-scheme: The \( \overline{\text{MS}} \)-mass \( m_{\overline{\text{MS}}} (\mu) \) is scale-dependent.

- The on-shell-scheme: The mass \( m_{\text{pole}} \) is defined as the pole of the propagator.

In perturbation theory one can convert between different schemes!
The precision and accuracy on the top mass

With increasing experimental precision, more theoretical issues become relevant:

- The top quark is neither stable nor a colour-singlet.
  - This re-introduces non-perturbative effects
  - The pole mass is ambiguous by an amount $O(\Lambda_{\text{QCD}})$.

- Experimentally measured are the decay products of the top.
  - For top measurements based on reconstruction:
    The $\overline{\text{MS}}$ mass distorts the Breit-Wigner shape.

Solution: Carefully define a short-distance mass, insensitive to non-perturbative effects, and relate the experimental measurement to this short-distance mass.

Fleming, Hoang, Mantry, Stewart, '07
Top-pair production

The leading-order Feynman diagrams:

The quark-antiquark channel dominates at the Tevatron, while the gluon-gluon channel dominates at the LHC.

We are interested in the inclusive process

\[ pp \rightarrow t\bar{t} + X \]

as well as the exclusive processes

\[ pp \rightarrow t\bar{t} + 0 \text{ jets}, \quad pp \rightarrow t\bar{t} + 1 \text{ jet}, \quad pp \rightarrow t\bar{t} + 2 \text{ jets}, \quad ... \]
Status of NLO calculations

- $pp \rightarrow t\bar{t}$
  - Nason, Dawson, Ellis, '88,
  - Beenakker, Kuijf, van Neerven, Smith, '89,

- $pp \rightarrow t\bar{t} + 1$ jet
  - Dittmaier, Uwer, S.W., '07,
  - Melnikov, Schulze, '10,

- $pp \rightarrow t\bar{t} + 2$ jets
  - Bevilaqua, Czakon, Papadopoulos, Worek, '10

- $pp \rightarrow b\bar{b}W^+W^-$
  - Bevilaqua, Czakon, van Hameren, Papadopoulos, Worek, '10
  - Denner, Dittmaier, Kallweit, Pozzorini, '10

- sub-process $pp \rightarrow t\bar{t} + b\bar{b}$
  - Bredenstein, Denner, Dittmaier, Pozzorini, '09,
  - Bevilaqua, Czakon, Papadopoulos, Pittau, Worek, '09
Numerical results on $t\bar{t}+$ jet production

Dependence of the cross section on renormalisation and factorisation scale:

Leading order is proportional to $\alpha_s^3$!

Tevatron:

\[ \sigma[pb] \]

Jet definition: $k_{\perp}$-algorithm with $R = 1$ applied to particles other than $t$ or $\bar{t}$.

LHC:

\[ \sigma[pb] \]

Status of NNLO calculations

The $\alpha_s^4$-correction to $pp \rightarrow t\bar{t}$ requires

- The **two-loop amplitudes** $gg \rightarrow t\bar{t}$, $q\bar{q} \rightarrow t\bar{t}$
  Czakon, Mitov, Moch,
  Bonciani, Ferroglia, Gehrmann, Maitre, von Manteufel, Studerus

- The **squared one-loop amplitudes** $gg \rightarrow t\bar{t}$, $q\bar{q} \rightarrow t\bar{t}$
  Kniehl, Körner, Merebashvili, Rogal, '08,
  Anastasiou, Aybat, '08

- A method to **handle the infrared divergences at NNLO**, in particular initial state partons and massive partons.
  Bierenbaum, Czakon, Mitov,
  Boughezal, Daleo, Gehrmann, Gehrmann-De Ridder, Luisoni, Monni, Ritzmann, Glover, Pires
Top pair production at NNLO

Total inclusive cross section for $t\bar{t}$ production now known at NNLO.

Czakon, Fiedler, Mitov, 2013

- Two-loop integrals computed numerically
- Infrared singularities treated by sector decomposition
- Result combined with NNLL resummation, theoretical uncertainty estimated to be $\sim 3\%$
In multi-scale problems there can be large logarithms in the perturbative expansion:

\[ \alpha_s^n \ln^j \beta \]

Top-pair production in the threshold region

\[ \beta = \sqrt{1 - \frac{4m_i^2}{s}} \]

⇒ sum large logarithms to improve perturbation theory

In addition, Coulomb singularities of the form

\[ \frac{1}{\beta^k} \]
NNLL resummation

Soft gluon resummation at next-to-next-to-leading logarithmic accuracy:
Moch, Uwer, ’08,
Czakon, Mitov, Sterman, ’09,
Kidonakis, ’10,
Ahrens, Ferroglia, Neubert, Pecjak, Yang, ’10,
Beneke, Falgari, Klein, Schwinn, ’11

Can re-expand NNLL to obtain an “approximate NNLO” result.

Differences between the various groups:

- Exact definition of the resummation variable.
- Resummation of soft gluons only or also Coulomb terms?
- Which scale dependent parts are included?
Parton showers are **exclusive and resum large logarithms** at LL accuracy.

**Avoid double-counting** when combining NLO calculations with parton showers: MC@NLO and POWHEG.

Frixione, Webber, ’02, Nason, ’04

Convenient tool: **POWHEG-BOX**

Alioli, Nason, Oleari, Re, ’10.

**Recent application to** $pp \rightarrow t\bar{t} + 1$ jet:

Kardos, Papadopoulos, Trocsanyi, ’11

Alioli, Moch, Uwer, ’11
Spin correlations

Within the Standard Model the top decays purely through left-handed weak decay.

In dilepton channel of top-pair production: Correlation between top and anti-top spins transferred to leptons.

\[
\frac{1}{\sigma d \cos \theta_l d \cos \theta_{\bar{l}}} = \frac{1}{4} (1 - C \cos \theta_l \cos \theta_{\bar{l}})
\]

\[
C(D0) = 0.1^{+0.45}_{-0.45}
\]

\[
C(CDF) = 0.6 \pm 0.5 \pm 0.2 \quad (EPS \ 2011)
\]

\[
C(Theory) = 0.777^{+0.027}_{-0.042}
\]

Mahlon, Parke, '97, Bernreuther, Brandenburg, Si, Uwer, '04
The forward-backward asymmetry at the Tevatron

Forward-backward or charge asymmetry in $q\bar{q} \rightarrow t\bar{t}(+\text{jets})$

**Origin**: Interference of $C$-odd with $C$-even parts.

$q\bar{q} \rightarrow t\bar{t}$: asymmetry appears first at NLO (Kühn, Rodrigo '98).

$A_{FB} @ NLO$ requires NNLO $p\bar{p} \rightarrow t\bar{t}$

$q\bar{q} \rightarrow t\bar{t} + \text{jet}$: asymmetry is LO effect (Halzen, Hoyer, Kim, '87, Bowen, Ellis, Rainwater, '05).

$A_{FB} @ NLO$ can be deduced from NLO $p\bar{p} \rightarrow t\bar{t} + \text{jet}$. 
The asymmetry for $t\bar{t}$ at the Tevatron

Lab frame:

$$A_{FB}(CDF) = (15.0 \pm 5.5)\%,$$
$$A_{FB}(Theory) = (5.8 \pm 0.9)\%,$$

$t\bar{t}$ rest frame:

$$A_{FB}(CDF) = (20.0 \pm 7.3)\%,$$
$$A_{FB}(D0) = (19.6 \pm 6.5)\%,$$
$$A_{FB}(Theory) = (8.7 \pm 1.0)\%,$$

**Theory status**: LO QCD (from NLO $p\bar{p} \rightarrow t\bar{t}$)

electroweak corrections small Hollik, Kollar, '07, Bernreuther, Si, '10, Kühn, Rodrigo, '11,
soft gluon corrections small Kidonakis, '11, Ahrens, Ferroglia, Neubert, Pecjak, Yang, '11

There is a certain tension ...
The forward-backward asymmetry in $t\bar{t} + \text{jet}$

$$p\bar{p} \rightarrow t\bar{t} + \text{jet} + X$$
$$\sqrt{s} = 1.96 \text{ TeV}$$
$$p_{T,\text{jet}} > 20\text{GeV}$$

$\sigma^\pm = \sigma(y_t > 0) \pm \sigma(y_t < 0)$,

$$A_{FB,LO}^t = \frac{\sigma^-_LO}{\sigma^+_{LO}}$$,

$$A_{FB,NLO}^t = \frac{\sigma^-_{LO}}{\sigma^+_{LO}} \left( 1 + \frac{\delta \sigma^-_{NLO}}{\sigma^-_{LO}} - \frac{\delta \sigma^+_{NLO}}{\sigma^+_{LO}} \right).$$

$$(\mu = \mu_{\text{ren}} = \mu_{\text{fact}})$$

- $A_{FB,LO}^t = O(\alpha_s^0)$, i.e. no dependence on $\mu_{\text{ren}}$
  mild dependence on $\mu_{\text{fact}} \ll$ theoretical uncertainty!

- $A_{FB,NLO}^t$ depends on $\mu_{\text{fact}}$ and $\mu_{\text{ren}}$
  asymmetry almost washed out by scale dependence.

Dittmaier, Uwer, S.W., '07, Melnikov, Schulze, '10
The charge asymmetry at the LHC

No forward-backward asymmetry in the lab frame at the LHC due to symmetric $pp$ initial state.

But: $t$ tend to follow initial $q$, while $\bar{t}$ tend to follow initial $\bar{q}$.

Initial $q$’s tend to have a larger momentum fraction (valence-like) than initial $\bar{q}$’s (always sea quarks).

⇒ Rapidity distribution of $t$ should be broader than the rapidity distribution of $\bar{t}$.

Complication: At the LHC $q\bar{q}$ initial state gives only a small fraction of the events, dominated by $gg$.

In view of the Tevatron results this measurement should be worth the effort !
The charge asymmetry at the LHC

Definition of the charge asymmetry:

\[ A_C = \frac{N(\Delta|y| > 0) - N(\Delta|y| < 0)}{N(\Delta|y| > 0) + N(\Delta|y| < 0)} \]

\[ \Delta|y| = |y_t| - |y\bar{t}| \]

LHC results:

\[ A_C(\text{ATLAS}) = (0.6 \pm 1.0)\% \]
\[ A_C(\text{CMS}) = (0.4 \pm 1.5)\% \]
\[ A_C(\text{Theory}) = (1.23 \pm 0.05)\% \]

Consistent with Standard Model!

Related observables with enhanced sensitivity: incline asymmetry

Berge, Westhoff, '13
Top quark can also be produced singly by an electroweak $Wtb$-vertex:

- **Flavour excitation**
- **W-gluon fusion**
- **s-channel**
- **Associated W-production**
Physics motivation for single top production:

- **Process sensitive to** $Wtb$ **vertex.**
  Non-standard couplings can give a hint on physics beyond SM.

- **Direct measurement of** CKM **matrix element** $V_{tb}$.  
  Verification of unitarity of CKM matrix.

- **The top quark is produced left-handed.**
  Since no hadronization occurs, spin correlations survive in the final decay products.

- The flavour excitation channel can be used to **extract the** $b$-quark density.
Status of NLO calculations

- **s-channel and t-channel:**
  Stelzer, Sullivan, Willenbrock, '98, Harris, Laenen, Phaf, Sullivan, S.W., '02, Cao, Schwienhorst, Yuan, '04, Campbell, Ellis, Tramontana, '04, Campbell, Frederix, Maltoni, Tramontano, '09, Falgari, Giannuzzi, Mellor, Signer, '11

- **Associated W-production:**
  Campbell, Tramontana, '05, Frixione, Laenen, Motylinski, Webber, White, '08

- **Resummation:**
  Kidonakis, '10, Zhu, Li, Wang, Zhang, '10

- **MC@NLO:**
  Frixione, Laenen, Motylinski, Webber, '05

- **POWHEG:**
  Alioli, Nason, Oleari, Re, '09
Status of NNLO calculations

- **t-channel:**
  “DIS-squared” calculation,
  neglecting colour-suppressed cross-talk
  between heavy- and light-quark line
  
  Brucherseifer, Caola, Melnikov, '14,
The CKM matrix element $V_{tb}$

$V_{tb}$ is known indirectly from unitarity

$$|V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2 = 1$$

to a very high precision: $|V_{tb}| = 0.9990 - 0.9993$

No way to measure $|V_{tb}|$ directly to this precision!

Sideremark: From top-pair production at the Tevatron:

$$\frac{BR(t \to Wb)}{BR(t \to Wq)} = \frac{|V_{tb}|^2}{|V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2} = 0.99^{+0.09}_{-0.08}$$

If we do not assume three generations, then it follows only

$$|V_{tb}| >> |V_{ts}|, |V_{td}|$$

Single top production: Direct measurement of $|V_{tb}|$ without any assumptions on the number of generations.
Spin correlations

At the electroweak $Wtb$-vertex the top quark is produced left-handed.

Since no hadronization occurs, spin correlations survive in the final decay products.

Of particular importance is the angle between the lepton $\bar{l}$ and the light-quark jet $\bar{q}'$.

Jezabek, Kühn, '94
Mahlon, Parke, '97
van der Heide, Laenen, Phaf, S.W., '00
Angular correlations

In W-gluon fusion or flavour excitation the top quark is highly polarized along the direction of the $\bar{d}$-quark. In addition the $u$-quark density is the largest among the quark densities. Look at the quantity

$$a = \frac{1}{2} (1 + \cos \theta_{ql})$$

where $\theta_{ql}$ is the angle between the light quark jet and the charged lepton.

In the rest frame of the top:

$$\frac{d\sigma}{da} = \sigma (2Pa + (1 - P)),$$

The slope of this distribution is given by

$$2P_{\text{signal}} \sigma_{\text{signal}} + 2P_{\text{background}} \sigma_{\text{background}}$$
Outlook: $t\bar{t}$ in association with multiple jets at NLO

LHC experiments measure top pair production in association with several jets.

NLO predictions available for $t\bar{t} + 0$ jets, $t\bar{t} + 1$ jet and $t\bar{t} + 2$ jets.

Higher jet multiplicities? Can use numerical approach:

$$\langle O \rangle^{\text{NLO real}}_{\text{real}} = \int_{n+1} \left( O_{n+1} d\sigma^R - O_n d\sigma^A \right)$$

$$\langle O \rangle^{\text{NLO virtual}}_{\text{virtual}} = \int_{n+\text{loop}} \left( O_n d\sigma_{\text{bare}}^V - O_n d\sigma^L \right)$$

$$\langle O \rangle^{\text{NLO insertion}}_{\text{insertion}} = \int_n \left( O_n d\sigma_{\text{CT}}^V + O_n \int_{\text{loop}} d\sigma^L + O_n \int_1 d\sigma^A + O_n d\sigma^C \right)$$

D. Mediger, J. Pečovnik, S.W., work in progress
Several two-loop integrals relevant to top physics are currently not known analytically. Which functions with which arguments appear in the evaluation of these integrals?

Polylogarithms:

\[ \text{Li}_m(x) = \sum_{j=1}^{\infty} \frac{x^j}{j^m} \]

Multiple polylogarithms:

\[ \text{Li}_{m_1,m_2,\ldots,m_k}(x_1,x_2,\ldots,x_k) = \sum_{j_1>j_2>\ldots>j_k>0} \frac{x_1^{j_1}}{j_1^{m_1}} \cdot \frac{x_2^{j_2}}{j_2^{m_2}} \cdot \ldots \cdot \frac{x_k^{j_k}}{j_k^{m_k}} \]

Elliptic polylogarithms:

\[ \text{ELi}_{n;m}(x;y;q) = \sum_{j=1}^{\infty} \sum_{k=1}^{\infty} \frac{x^j y^k}{j^n k^m q^{jk}} \]

S. Bloch, P. Vanhove, ’13; L. Adams, Ch. Bogner, S.W., ’14
Summary

- **Top physics** is a very active field in theory.
- **Top mass** is very close to the electroweak symmetry breaking scale.
- The value of the **top mass** is essential for many precision measurements.
- **Angular distributions** are very interesting:
  - Spin correlations in top-pair production and single top.
  - Charge asymmetry at Tevatron and LHC.