When protons collide: Multiparton interactions in QCD

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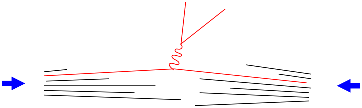
Hadron-hadron collisions

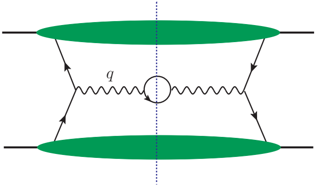
- standard description based on factorization formulae

  cross sect = parton distributions × parton-level cross sect

  example: Z production

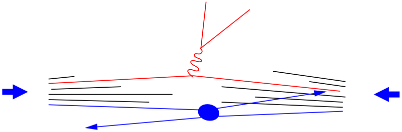
  $pp \to Z + X \to \ell^+\ell^- + X$





- factorization formulae are for inclusive cross sections $pp \to Y + X$
  where Y = produced in parton-level scattering, specified in detail
  X = summed over, no details
- have also interactions between "spectator" partons
  but their effects cancel in inclusive cross sections thanks to unitarity

Multiparton interactions

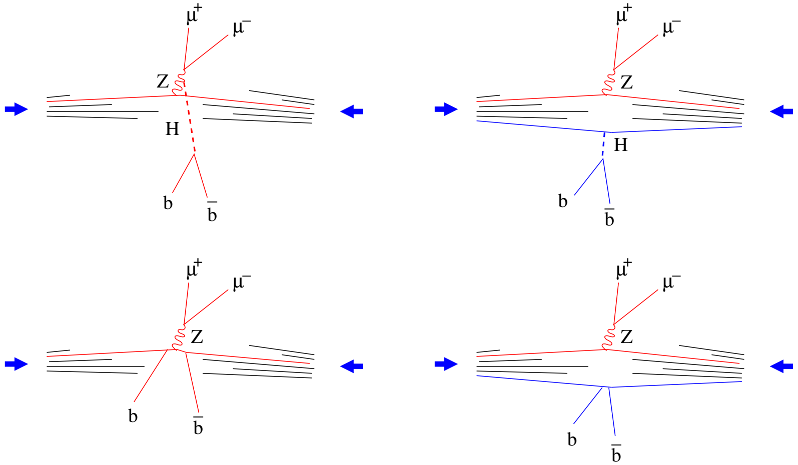


- generically take place in hadron-hadron collisions
  at high c.m. energy several interactions can be hard
- effects cancel or are suppressed in sufficiently inclusive quantities
  but do affect final state properties
- many studies:
  theory: Treleani et al; Artru, Mekhfi; recent activity
  experiment: AFS, UA2, Tevatron, first results from LHC
  Monte Carlo generators: Pythia, Herwig++, Sherpa
- expected to be important for many processes at LHC
  see e.g. Procs. of Multi-Parton Interactions at the LHC (arXiv:1111.0469)
  Procs. of MPI 2011 (to appear shortly)
  significant effect found in $pp \to W + 2$ jets $+ X$, ATLAS-CONF-2011-160

Relevance for LHC

example: $pp \to H + Z \to b\bar{b} + Z$ Del Fabbro, Treleani 1999

- multiple interactions contribute to signal and background



Multiparton interactions

- phenomenology based on simple, physically intuitive formula

  cross sect = multiparton distributions × parton-level cross sect's

  and ansatz

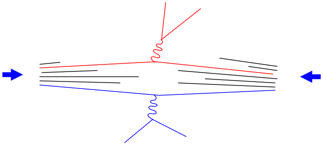
  multiparton distribution = factor × $\prod$ single-parton distributions

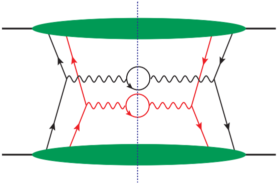
  Paver, Treleani 1982, 1984; Mekhfi 1985, ...

  also underlies implementation in event generators
- questions:
  - to which extent can these formulae be derived in QCD?
  - where and how do they need to be modified?
  - can factorization theorems for multiparton interactions be formulated and proven?
- no definitive answers to all points, but some results and identified problems MD, Ostermeier, Schäfer 2011
- ultimate goal: improved theory as a guide for phenomenology

Theoretical framework

- require all interactions to have hard scale
  $\rightsquigarrow$ predictive power from factorization and perturb. theory
  extrapolation to low scales $\rightarrow$ model for underlying event
- consider gauge boson pair production (pairs of γ^*, W, Z)



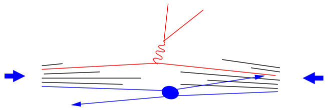


- keep transverse momentum of bosons observed
  - are interested in final-state distributions
  - allows investigation of Sudakov logarithms
    $\rightarrow$ basis for parton showers in event generators
  - need k_T dependent parton distributions
    build on theory for single Drell-Yan process

Collins, Soper 1982; . . . ; Collins 2011

Multijet production

- multiple jet production of high practical importance, but

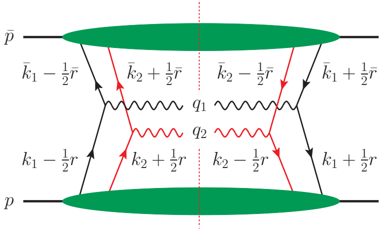


- single hard scattering: factorization with k_T dependent dist's for prod'n of colorless particle (γ^*, Z, W, Higgs, ...)
- but serious problems for colored particles (jets, heavy quarks, ...) due to gluon exchange between spectator partons and produced jets
  Mulders, Rogers 2010
- $\rightsquigarrow$ roadblock for developing full theory for multijets
  but at tree level can readily adapt results from boson prod'n

correspondence of variables:

	two e.w. bosons	two dijets
large	boson invariant mass	dijet inv. mass / jet p_T
small	boson tranv. mom.	sum of transv. momenta in dijet

Basic results: space-time structure

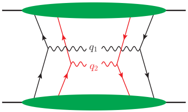


momentum conservation $\Rightarrow r + \bar{r} = 0$

- longitudinal parton momenta $x_i p$, $\bar{x}_i \bar{p}$ fixed by final state
  exactly as for single hard scattering
- transverse parton momenta not the same
  in amplitude and conjugate amplitude
- Fourier transform to impact parameter: $\boldsymbol{r} \to \boldsymbol{y}$ and $\bar{\boldsymbol{r}} \to \bar{\boldsymbol{y}}$
  $\boldsymbol{r} + \bar{\boldsymbol{r}} = \boldsymbol{0}$ implies $\boldsymbol{y} = \bar{\boldsymbol{y}}$
- interpretation: $\boldsymbol{y}$ = transv. dist. between two scattering partons
  equal in both colliding protons

same procedure for partons with index 2

Basic structure: cross section



- get cross section formula

$$\frac{d\sigma}{dx_1\, d\bar{x}_1\, d^2\boldsymbol{q}_1\;\, dx_2\, d\bar{x}_2\, d^2\boldsymbol{q}_2} = \left[\prod_{i=1}^{2} \hat{\sigma}_i\big(q_i^2 = x_i \bar{x}_i s\big)\right]$$

$$\times \left[\prod_{i=1}^{2} \int d^2\boldsymbol{k}_i\, d^2\bar{\boldsymbol{k}}_i\, \delta(\boldsymbol{q}_i - \boldsymbol{k}_i - \bar{\boldsymbol{k}}_i)\right] \int d^2\boldsymbol{y}\, F(x_i, \boldsymbol{k}_i, \boldsymbol{y})\, F(\bar{x}_i, \bar{\boldsymbol{k}}_i, \boldsymbol{y})$$

$\hat{\sigma}_i =$ parton-level cross section

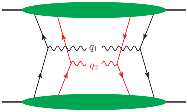
$F(x_i, \boldsymbol{k}_i, \boldsymbol{y}) =$ k_T dependent two-parton distribution

- result follows from Feynman graphs and hard-scattering approximation
  no semi-classical approximation required
- $\int d^2\boldsymbol{q}_1 \int d^2\boldsymbol{q}_2$ in cross sect. $\rightarrow$ k_T integrated distributions

$$F(x_i, \boldsymbol{y}) = \int d^2\boldsymbol{k}_1 \int d^2\boldsymbol{k}_2\, F(x_i, \boldsymbol{k}_i, \boldsymbol{y})$$

recover usual cross section formula (slide 5)

Multiparton distributions



- can define as operator matrix element
  (like for single-parton densities)

$$F(x_i, \boldsymbol{k}_i, \boldsymbol{y}) = \underset{z_i \to (x_i, \boldsymbol{k}_i)}{\mathcal{FT}} \langle p | \bar{q}\big(-\tfrac{1}{2} z_2\big) \Gamma_2\, q\big(\tfrac{1}{2} z_2\big)\; \bar{q}\big(y - \tfrac{1}{2} z_1\big) \Gamma_1\, q\big(y + \tfrac{1}{2} z_1\big) | p \rangle$$

  - essential for studying factorization, scale evolution, etc.
  - possibility for lattice calculations

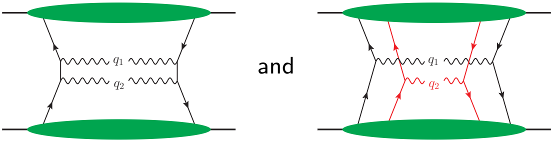
Power behavior: single versus double hard scattering

- from scattering formulae readily find

$$s \frac{d\sigma}{dx_1\, d\bar{x}_1\, d^2\boldsymbol{q}_1\;\, dx_2\, d\bar{x}_2\, d^2\boldsymbol{q}_2} \sim \frac{1}{Q^2\Lambda^2}$$

$Q^2 \sim q_i^2,\ \Lambda^2 \sim \mathrm{GeV}$

for both and



$\Rightarrow$ double scattering not power suppressed

Power behavior: single versus double hard scattering

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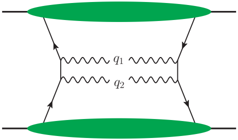
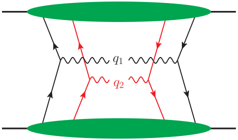
for both  and 

$\Rightarrow$ double scattering not power suppressed

- but if integrate over $\boldsymbol{q}_1$ and $\boldsymbol{q}_2$ then

single: $$s \, \frac{d\sigma}{dx_1 \, d\bar{x}_1 \; dx_2 \, d\bar{x}_2} \sim 1$$ since $\int d^2(\boldsymbol{q}_1 + \boldsymbol{q}_2) \sim \Lambda^2$ and $\int d^2(\boldsymbol{q}_1 - \boldsymbol{q}_2) \sim Q^2$

double: $$s \, \frac{d\sigma}{dx_1 \, d\bar{x}_1 \; dx_2 \, d\bar{x}_2} \sim \frac{\Lambda^2}{Q^2}$$ since $\int d^2\boldsymbol{q}_1 \int d^2\boldsymbol{q}_2 \sim \Lambda^4$

i.e. single hard scattering has larger phase space for transv. momenta

Power behavior: single versus double hard scattering

- from scattering formulae readily find

$$s \frac{d\sigma}{dx_1\, d\bar{x}_1\, d^2\boldsymbol{q}_1\; dx_2\, d\bar{x}_2\, d^2\boldsymbol{q}_2} \sim \frac{1}{Q^2\Lambda^2}$$

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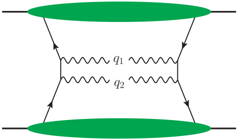
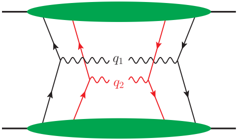
for both  and 

$\Rightarrow$ double scattering not power suppressed

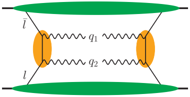
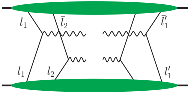
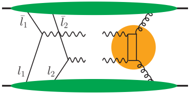
- if integrate only over $\boldsymbol{q}_1 + \boldsymbol{q}_2$ then no power suppression yet

$$s \frac{d\sigma}{dx_1\, d\bar{x}_1\, dx_2\, d\bar{x}_2\, d^2(\boldsymbol{q}_1 - \boldsymbol{q}_2)} \sim \frac{1}{Q^2}$$

Situation so far

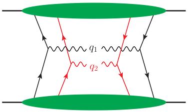
- have basic cross section formula
  for observed transv. momenta in final state
- but have glossed over important details → next slides

Interference

	$\dfrac{s\,d\sigma}{\prod_{i=1}^{2} dx_i\, d\bar{x}_i\, d^2\boldsymbol{q}_i}$	$\dfrac{s\,d\sigma}{\prod_{i=1}^{2} dx_i\, d\bar{x}_i}$
	$\dfrac{1}{\Lambda^2 Q^2}$	1
	$\dfrac{1}{\Lambda^2 Q^2}$	$\dfrac{\Lambda^2}{Q^2}$
	$\dfrac{1}{\Lambda^2 Q^2}$	$\dfrac{\Lambda^2}{Q^2}$

- interference between single and double scattering:
  - leading power when differential in $\boldsymbol{q}_i$
  - power suppressed when $\int d^2\boldsymbol{q}_i$, twist-three parton distributions
- at small $x_1 \sim x_2 \sim x$ expect
  - single scattering $\propto x^{-\lambda}$ with $x f(x) \sim x^{-\lambda}$
  - double scattering $\propto x^{-2\lambda}$
  - interference ? how do twist-three distributions behave for small x ?

Spin structure

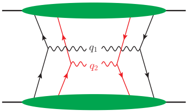


$$F(x_i, \boldsymbol{k}_i, \boldsymbol{y}) = \underset{z_i \to (x_i, \boldsymbol{k}_i)}{\mathcal{FT}} \langle p|\bar{q}\big(-\tfrac{1}{2}z_2\big)\Gamma_2\, q\big(\tfrac{1}{2}z_2\big)\, \bar{q}\big(y - \tfrac{1}{2}z_1\big)\Gamma_1\, q\big(y + \tfrac{1}{2}z_1\big)|p\rangle$$

- at leading twist: $\Gamma_i = \frac{1}{2}\gamma^+,\ \frac{1}{2}\gamma^+\gamma_5,\ \frac{1}{2}i\sigma^{+\alpha}\gamma_5$
  $\Leftrightarrow$ unpolarized, long. polarized, transv. polarized quarks
  similar classification for gluons
- spin correlations even in unpolarized target, e.g.
  $$\Gamma_1 = \Gamma_2 = \tfrac{1}{2}\gamma^+\gamma_5 \quad \Leftrightarrow \quad q_1^\uparrow q_2^\uparrow + q_1^\downarrow q_2^\downarrow - q_1^\uparrow q_2^\downarrow - q_1^\downarrow q_2^\uparrow$$
  not suppressed by hard scattering in two-boson prod'n
  $\rightsquigarrow$ affects rate
- transverse spin correlations from $\Gamma_1 = \Gamma_2 = \frac{1}{2}i\sigma^{+\alpha}\gamma_5$
  $\rightsquigarrow$ $\cos 2\phi$ modulation between decay planes of the two bosons
  at low $\boldsymbol{q}_1, \boldsymbol{q}_2$ single hard scattering does not give $\cos 2\phi$ term
  in general: correlated scattering planes

MD, Kasemets 2012

Spin structure



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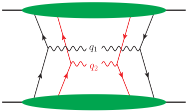
  not suppressed by hard scattering in two-boson prod'n
  $\rightsquigarrow$ affects rate
- could naively expect spin effects to decrease for small x_1, x_2, but
  - will see counter-example on slide 27
  - for $x_1 \sim x_2 \ll 1$ spin correlations may be
    weak between a parton and the proton (far away in rapidity)
    important between two partons (close in rapidity)

Color structure



$$F(x_i, \boldsymbol{k}_i, \boldsymbol{y}) = \underset{z_i \to (x_i, \boldsymbol{k}_i)}{\mathcal{FT}} \langle p| \bar{q}\big(-\tfrac{1}{2} z_2\big) \Gamma_2\, q\big(\tfrac{1}{2} z_2\big)\, \bar{q}\big(y - \tfrac{1}{2} z_1\big) \Gamma_1\, q\big(y + \tfrac{1}{2} z_1\big) |p\rangle$$

- operators $\bar{q}_2\, q_2$ and $\bar{q}_1\, q_1$ can couple to color singlet or octet:

$$^1F \to (\bar{q}_2 \mathbb{1} q_2)\, (\bar{q}_1 \mathbb{1} q_1) \qquad\qquad ^8F \to (\bar{q}_2 t^a q_2)\, (\bar{q}_1 t^a q_1)$$

- in gauge boson pair production contrib's from $^1F\,^1F$ and $^8F\,^8F$
- color octet distributions essentially unknown
  (no probability interpretation as a guide)
  suppressed by Sudakov logarithms, but not necessarily negligible
  Mekhfi 1988; Manohar, Waalewijn 2011
- for two-gluon dist's more color structures: 1, 8_S, 8_A, 10, $\overline{10}$, 27

spin and color correlations discussed by Mekhfi 1985
but not followed up in literature until 2011
not implemented in phenomenology

Situation so far

- can extend basic cross section formula to include contributions from
  - spin correlations
    direct influence on rates and distributions
  - color correlations
  - interference between single and multiple scattering
- many distributions needed to calculate multiparton scattering
  some of them qualitatively unknown

such effects are not included in phenomenology or event generators

Approximation by single-parton distributions

$$F(x_i, \boldsymbol{k}_i, \boldsymbol{y}) = \underset{z_i \to (x_i, \boldsymbol{k}_i)}{\mathcal{FT}} \langle p | \bar{q}\big(-\tfrac{1}{2} z_2\big) \Gamma_2\, q\big(\tfrac{1}{2} z_2\big)\, \bar{q}\big(y - \tfrac{1}{2} z_1\big) \Gamma_1\, q\big(y + \tfrac{1}{2} z_1\big) | p \rangle$$

- between $\bar{q}_2\, q_2$ and $\bar{q}_1\, q_1$ insert complete set $\sum_X |X\rangle\langle X|$ of states
- if assume that single-proton states $|p\rangle\langle p|$ dominate in $\sum |X\rangle\langle X|$
  then $F(x_i, \boldsymbol{k}_i, \boldsymbol{y}_1) \approx$ product of single-quark distributions

  $$\langle p|\bar{q}_2\, q_2\ \bar{q}_1\, q_1|p\rangle \approx \sum_{p'} \langle p|\bar{q}_2\, q_2|p'\rangle\, \langle p'|\bar{q}_1\, q_1|p\rangle$$

  in physical terms: neglect correlations between parton 1 and 2
- transv. momenta $\boldsymbol{p}$ and $\boldsymbol{p}'$ differ $\rightsquigarrow$ generalized parton distributions
  - appear in exclusive processes, e.g. $\gamma p \to J/\Psi\, p$ measured at HERA
  - Fourier trf. from $\boldsymbol{p} - \boldsymbol{p}'$ to impact parameter $\rightsquigarrow$ joint dist'n of partons in long. mom. and transv. position

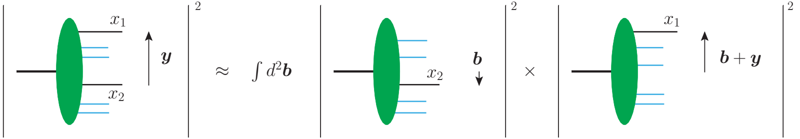
Approximation by single-parton distributions

$$\langle p|\bar{q}_2\, q_2\;\bar{q}_1\, q_1|p\rangle \approx \sum_{p'} \langle p|\bar{q}_2\, q_2|p'\rangle\,\langle p'|\bar{q}_1\, q_1|p\rangle$$

- relation is approximate but
  - multiparton cross section $\propto \int d^2\boldsymbol{y}\, F(x_i, \boldsymbol{k}_i, \boldsymbol{y})\, F(\bar{x}_i, \bar{\boldsymbol{k}}_i, \boldsymbol{y})$
    $\rightsquigarrow$ $\boldsymbol{y}$ is unobservable
  - in generalized parton dist's $\boldsymbol{p} - \boldsymbol{p}'$ is measurable
- especially simple for collinear distributions:

$$F(x_i, \boldsymbol{y}) \;\approx\; \int d^2\boldsymbol{b}\; f(x_2, \boldsymbol{b})\, f(x_1, \boldsymbol{b}+\boldsymbol{y})$$

with $f(x, \boldsymbol{b}) =$ impact-parameter distribution



used for long time in literature, based on geometric intuition

Correlations between x and $\boldsymbol{b}$

- if assume $f(x,\boldsymbol{b}) = f(x)\, F(\boldsymbol{b})$ then

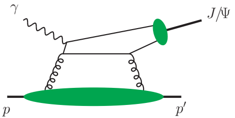
$$F(x_i, \boldsymbol{y}) \;\approx\; \int d^2\boldsymbol{b}\; f(x_2, \boldsymbol{b})\, f(x_1, \boldsymbol{b}+\boldsymbol{y}) = f(x_2)\, f(x_1)\, G(\boldsymbol{y})$$

  used in phenomenology and event generators

  with adjustments at high x to ensure mom. conservation

- HERA results on $\gamma p \to J/\Psi\, p$
  $\to$ correlation between x and
  $\boldsymbol{b}$ dist'n for small-x gluons

$$\langle \boldsymbol{b}^2 \rangle \propto \text{const} + 4\alpha' \log \frac{1}{x}$$
  with $\alpha' \approx 0.15\,\text{GeV}^{-2}$



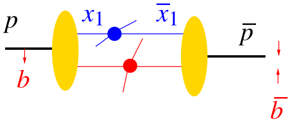
- lattice calculations $\to$ even stronger drop of $\langle \boldsymbol{b}^2 \rangle$ for quarks at high x

Consequence for multiple interactions

- have indications for decrease of $\langle \boldsymbol{b}^2 \rangle$ with x
- if interaction 1 produces high-mass system
  $\rightarrow$ have large $x_1, \bar{x}_1$
  $\rightarrow$ smaller impact parameters $b, \bar{b}$
  $\rightarrow$ collision more central
  $\rightarrow$ secondary interactions enhanced

  Frankfurt, Strikman, Weiss 2003

  study in framework of Pythia: Corke, Sjöstrand 2011



Spin and color correlations

- for $x_i \gtrsim 0.1$ may estimate $F(x_i, \boldsymbol{k}_i, \boldsymbol{y})$ using quark models
  expect large correlation effects; for three quarks have e.g.
  - color structure: $\epsilon^{ijk}\epsilon^{i'j'k} = \delta^{ii'}\delta^{jj'} - \delta^{ij'}\delta^{i'j}$
    $\rightsquigarrow$ $\left({}^{8}F\right) = -\sqrt{2}\left({}^{1}F\right)$
  - with $SU(6)$ symmetric wave fct. get

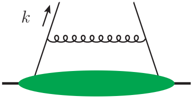
$$\Delta u/u = 2/3 \qquad F_{\Delta u,\Delta u}/F_{u,u} = 1/3$$
$$\Delta d/d = -1/3 \qquad F_{\Delta u,\Delta d}/F_{u,d} = -2/3$$

- take these as input for evolution to higher scales
  - do correlations "evolve away"? if yes, how fast?
  - also gives distributions at lower x_i
    partons radiate partons → lose momentum
    but misses sea quarks and gluons of nonperturbative origin
    PDF fits by Reya et al. require $\bar{q}$ and g even at low scales

work in progress

High q_T: more predictive power

- consider region $\Lambda \ll q_T \ll Q$, with $q_T \sim |\boldsymbol{q}_i|$ have $|\boldsymbol{k}_i| \sim q_T$
- k_T dependent dist'n = hard scattering $\otimes$ collinear dist'n
  hard scattering closely related to DGLAP splitting functions

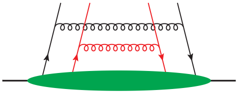


k_T integrated distribution essentially is

$$f(x;\mu^2) = \pi \int_0^{\mu^2} d\boldsymbol{k}^2\, f(x,\boldsymbol{k})$$

High q_T: more predictive power

- consider region $\Lambda \ll q_T \ll Q$, with $q_T \sim |\boldsymbol{q}_i|$ have $|\boldsymbol{k}_i| \sim q_T$
- k_T dependent dist'n = hard scattering $\otimes$ collinear dist'n
  hard scattering closely related to DGLAP splitting functions
- ladder graphs: independent hard scatters for pair 1 and 2



- $|\boldsymbol{y}|$ of hadronic size
- color factors favor singlet dist's compared to octet ones

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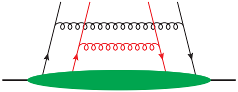

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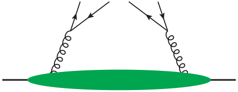
- splitting graphs



- perturbatively small $|\boldsymbol{y}|$
- longitudinal q and $\bar{q}$ spins fully correlated
  also large transv. spin correlation

- ladder graphs power suppressed by Λ^2/q_T^2 compared with splitting but have small-x enhancement

Evolution of collinear distributions

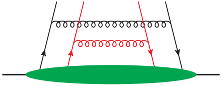
consider only color singlet combination F_1, situation for F_8 more complicated

$$F(x_i, \boldsymbol{k}_i, \boldsymbol{y}) = \underset{z_i \to (x_i, \boldsymbol{k}_i)}{\mathcal{FT}} \langle p | \bar{q}\big(-\tfrac{1}{2} z_2\big) \Gamma_2\, q\big(\tfrac{1}{2} z_2\big)\; \bar{q}\big(y - \tfrac{1}{2} z_1\big) \Gamma_1\, q\big(y + \tfrac{1}{2} z_1\big) | p \rangle$$

- $F(x_i, \boldsymbol{y})$ for $\boldsymbol{y} \neq \boldsymbol{0}$:

  separate DGLAP evolution for pair 1 and 2

  $$\frac{d}{d \log \mu} F(x_i, \boldsymbol{y}) = P \otimes_{x_1} F + P \otimes_{x_2} F$$



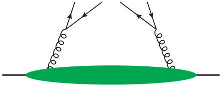
- $\int d^2\boldsymbol{y}\, F(x_i, \boldsymbol{y})$:

  extra term from $2 \to 4$ parton transition

  since $F(x_i, \boldsymbol{y}) \sim 1/\boldsymbol{y}^2$

  Kirschner 1979; Shelest, Snigirev, Zinovev 1982

  Gaunt, Stirling 2009

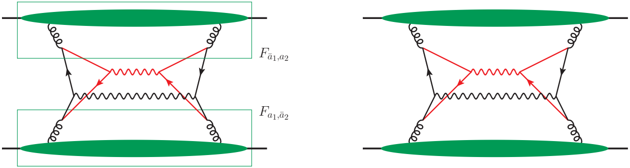


- $\rightsquigarrow$ consistency problem for frequently made ansatz

  $$F(x_i, \boldsymbol{y}; \mu) = G(\boldsymbol{y}) \left[\int d^2\boldsymbol{y}\, F(x_i, \boldsymbol{y}) \right]_\mu$$

  if on r.h.s. include $2 \to 4$ term in evolution

Deeper problems with the splitting graphs



- contribution from splitting graphs in cross section gives divergent integrals $\int d^2\boldsymbol{y}\, F(x_i, \boldsymbol{k}_i, \boldsymbol{y})\, F(\bar{x}_i, \bar{\boldsymbol{k}}_i, \boldsymbol{y})$
- double counting problem between double scattering with splitting and single scattering at loop level

  same problem for jets: Cacciari, Salam, Sapeta 2009
- possible solution:
  subtract splitting contribution from two-parton dist's when $\boldsymbol{y}$ is small
  will also modify their scale evolution; remains to be worked out

Summary

- multiple hard interactions not power suppressed
  for cross section differential in transverse momenta
- nontrivial spin and color structure
  interference between single and multiple scattering
  size of these effects presently unknown
  $\rightsquigarrow$ extra uncertainty in quantitative description
  in addition to uncertainty on "usual" multiparton dist's
- some simplification for transv. mom. $|\boldsymbol{q}_i| \gg \Lambda$
- multi-parton dist's depend on transverse distance $\boldsymbol{y}$ between partons
  correlations with x have consequences on mult. int. rate
- should remove small $\boldsymbol{y}$ contribution in order to avoid divergences
  and double counting
- opportunity at LHC:
  investigate in detail final states susceptible to mult. int.
  $\rightsquigarrow$ help to sort out which effects are important and which are not