Light dark stuff in cosmology

Warm dark matter, hot dark matter and dark radiation

Jan Hamann

Graduiertenkolleg seminar
@ Universität Freiburg

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What is the Universe made of?

Assuming the $\Lambda$CDM-model:

NASA's cosmic pie

- Dark Energy: 72%
- Dark Matter: 23%
- Atoms: 4.6%

today (z = 0)
What is the Universe made of?

Assuming the ΛCDM-model:

NASA's cosmic pie

- Dark Energy: 72%
- Dark Matter: 23%
- Atoms: 4.6%
- Neutrinos: 0.1 - 2%
- Photons: 0.006%

today (z = 0)
Going back in time/redshift...

\[ \log \rho \]

Today

\[ \log(1+z) = \log \frac{1}{a} \]

(Dark energy \( \rho = \text{const.} (?) \)

(log(redshift/scale factor))
Going back in time/redshift...

\[ \log(1+z) = \log \frac{1}{a} \]

log \( \rho \) (energy density)

non-relativistic matter
\[ \rho \sim \frac{1}{a^3} \]

Dark energy
\[ \rho = \text{const.} \quad (?) \]

log(1+z) = \log \frac{1}{a} (redshift/scale factor)
Going back in time/redshift...

\[ \log(1+z) = \log \frac{1}{a} \]

- Dark energy: \( \rho = \text{const. (?)} \)
- Relativistic matter (radiation): \( \rho \sim \frac{1}{a^4} \)
- Non-relativistic matter: \( \rho \sim \frac{1}{a^3} \)

\( \rho \) (energy density)

log(1+z) = log 1/a (redshift/scale factor)

today
Cosmological observables: Low redshift probes

\[ \log(1+z) = \log \frac{1}{a} \]

(log redshift/scale factor)

\[ \log \rho \] (energy density)

Galaxy surveys

Weak lensing

Type Ia supernovae

\[ z \sim O(1) \]

today

\[ \log(1+z) = \log \frac{1}{a} \] (redshift/scale factor)
Cosmological observables: Decoupling era

Cosmic Microwave Background temperature and polarisation anisotropies

$\log(1+z) = \log \frac{1}{a}$

$\log \rho$ (energy density)

$z \sim O(10^3)$

$z = 1100$

today

log(1+z) = log 1/a (redshift/scale factor)
Cosmological observables:
Big Bang Nucleosynthesis

$\log(1+z) = \log \frac{1}{a}$

(redshift/scale factor)

$\log \rho$ (energy density)

Primordial abundances of light elements

$z \sim O(10^8-10^9)$

log(1+z) = log 1/a (redshift/scale factor)
# Matter content:
## a cosmologist's view

<table>
<thead>
<tr>
<th></th>
<th>interacting</th>
<th>non-interacting</th>
</tr>
</thead>
<tbody>
<tr>
<td>relativistic p &gt;&gt; m</td>
<td>Photons (CMB)</td>
<td>Dark radiation</td>
</tr>
<tr>
<td>non-relativistic p &lt;&lt; m</td>
<td>Baryons</td>
<td>Dark matter</td>
</tr>
</tbody>
</table>
Classifying the dark sector: free streaming

- Once the dark stuff is decoupled, it can stream freely.
- Depending on their velocity, particles have a finite horizon (for $v=c$, it is equal to the particle horizon, $d_H \sim H^{-1}$).
- Important effect on the growth of density perturbations.
Classifying the dark sector: free streaming

- Slow particles (or large scale perturbation)

Gravitational potential

Remains confined in potential well
Classifying the dark sector: free streaming

- Fast particles (or small scale perturbation)

Gravitational potential

Can escape from potential well

Perturbation gets washed out
Classifying the dark sector: free streaming

- Free streaming wavenumber

\[ k_{fs}(t) = \left( \frac{4\pi G \bar{\rho}(t) a^2(t)}{\langle p/m \rangle^2} \right)^{1/2} \]

- Free streaming length

\[ \lambda_{fs} \equiv \frac{2\pi}{k_{fs}} = 2\pi \sqrt{\frac{2}{3} \frac{\sqrt{\langle p/m \rangle^2}}{a(t)H(t)}} \]

→ clustering for \( k \ll k_{fs}, \lambda \gg \lambda_{fs} \)
non-clustering for \( k \gg k_{fs}, \lambda \ll \lambda_{fs} \)

depends on particle rest mass \( m \) and momentum distribution \( f(p) \)
Classifying the dark sector: free streaming

Based on the free-streaming length at matter-radiation equality, we can define:

- \( \lambda_{fs} \rightarrow 0 \) Cold dark matter
- \( O(10 \text{ kpc}) \geq \lambda_{fs} \geq O(10 \text{ pc}) \) Warm dark matter
- \( \lambda_{fs} \geq O(10 \text{ kpc}) \) Hot dark matter
- \( \lambda_{fs} \rightarrow H_0^{-1} \) Dark radiation

All evaluated at \( z_{eq} \)

\[ \Omega_r(z_{eq}) = \Omega_m(z_{eq}) \]
Cold, warm, hot or radiation?

- Structures do not form at all if we only have dark radiation → need dark matter
- Structure formation appears to be bottom-up
  - Galaxies existed before clusters
    → DM cannot be hot (at least not all of it)
- Some aspects of galactic structure may actually prefer a degree of “warmness” (cusp, satellites)
- Nonetheless: minimal cosmological standard model
  \[ \Lambda CDM = \text{Cold Dark Matter} + 3 \text{ massless neutrinos} \]
Beyond $\Lambda$CDM

I. CDM + hot dark matter
   $\rightarrow$ massive neutrinos

II. CDM + dark radiation
    $\rightarrow$ massless neutrinos + additional relativistic stuff

III. CDM + hot dark matter + dark radiation
    $\rightarrow$ massless neutrinos + massive sterile neutrinos
I. Massive neutrinos as hot dark matter
The cosmic neutrino background

- $T \sim 1$ MeV: neutrinos decouple from the plasma
- $T \sim 0.2$ MeV: $e^+e^-$-annihilation
  $\rightarrow$ heats up photon background, but not the neutrino background

\[
T_\nu = \left( \frac{4}{11} \right)^{1/3} T_\gamma \simeq 1.95 \text{ K}
\]
The cosmic neutrino background

- Number density (per flavour):
  \[ n_\nu = \frac{3}{2} \frac{\zeta(3)}{\pi^2} T_\nu^3 \]

- For \( T_\nu \ll m_\nu \), neutrino energy density is given by
  \[ \rho_\nu = \sum n_\nu m_\nu \]
  \[ \rightarrow \quad \Omega_\nu h^2 = \frac{\sum m_\nu}{93 \text{ eV}} \]
Laboratory constraints on neutrino masses

- Lower limit from neutrino oscillation data
- Upper limit from Tritium beta decay

\[ 0.05 \text{ eV} < \sum m_\nu < 7 \text{ eV} \]

\[ \rightarrow 0.001 < \Omega_\nu < 0.12 \]

- Since \( \Omega_{dm} \sim 0.23 \), neutrinos can at most make up a fraction of the total dark matter density
Structure formation
with massive neutrinos

$\Sigma m_\nu = 0\ eV$
$\Sigma m_\nu = 7\ eV$

[N-body simulations and movie by T. Haugbølle]
The matter power spectrum

\[ \Delta P/P \approx 8 \frac{\Omega_\nu}{\Omega_m} \]

Wavelength, \( \lambda [h^{-1} \text{ Mpc}] \)

Power spectrum, \( P(k) \)

Wavenumber, \( k [h \text{ Mpc}^{-1}] \)

\( \Sigma m_\nu = 0.0 \text{ eV} \)
\( \Sigma m_\nu = 1 \times 1.2 \text{ eV} \)

same total matter density
The matter power spectrum

CMB

Weak lensing

Lyman-α forest

CMB is also sensitive to the neutrino mass through its impact on the expansion rate.
The matter power spectrum

Amplitude of density perturbations \( \sim O(1) \) \( \rightarrow \) linear perturbation theory breaks down

"Non-linear" regime

CMB, galaxy surveys, weak lensing, Lyman-\( \alpha \) forest
Neutrino mass constraints

95%-credible upper limits

- WMAP only, ΛCDM+mν
  [Komatsu et al. (2010)]

- WMAP + small scale CMB + SDSS power spectrum + SN Ia + HST,
  range reflects varying complexity of models
  [Hannestad et al. (2010), Gonzalez-Garcia et al. (2010), De Putter et al. (2011)]
Neutrino mass constraints

Predicted 95%-sensitivities

Planck 2013
[Perotto et al. (2006)]

Planck + Euclid (WL + galaxy power spectrum) 2020+
[JH et al. (in preparation)]
II. Dark radiation
Cosmic Neutrino Background

Neutrino energy density:

\[ \rho_\nu^{\text{act}} = 3 \cdot \frac{g_\nu}{(2\pi)^3} \int d^3 q \, q f_\nu(q) = N_{\text{eff}}^{\text{act}} \cdot \frac{7\pi^2}{120} \left(\frac{4}{11}\right)^{4/3} T_\gamma^4 \]

LEP: 2.984 ± 0.008

Large mixing ensures that different mass/flavour eigenstates typically share a common momentum distribution

[Dolgov et al. (2002), Wong (2002)]
Cosmic Neutrino Background

Neutrino energy density:

\[ \rho_{\nu}^{\text{act}} = 3 \cdot \frac{g_\nu}{(2\pi)^3} \int d^3q \ q f_\nu(q) = N_{\text{eff}}^{\text{act}} \cdot \frac{7\pi^2}{120} \left( \frac{4}{11} \right)^{4/3} T_\gamma^4 \]

For \( f_\nu = f_{\text{FD}} \), one would have \( N_{\text{eff}}^{\text{act}} = 3 \)

Small correction due to \( \nu_e \)s not being quite completely decoupled at \( e^+e^- \)-annihilation + QED correction

\[ N_{\text{eff}}^{\text{act}} = 3.046 \quad \text{[Mangano et al. (2005)]} \]

Standard Model expectation:
Radiation content of the Universe

Other light stuff? *(Dark radiation)*

\[ \rho_x = N_x \cdot \frac{7\pi^2}{120} \left( \frac{4}{11} \right)^{4/3} T_\gamma^4 \]
Radiation content of the Universe

Other light stuff? (*Dark radiation*)

\[
\rho_X = N_X \cdot \frac{7\pi^2}{120} \left( \frac{4}{11} \right)^{4/3} T_\gamma^4
\]

Putting it all together:

\[
\rho_r = \rho_\gamma + \rho_\nu^{\text{act}} + \rho_X
\]

\[
= \frac{\pi^2}{15} T_\gamma^4 \left[ 1 + \left( N_{\text{eff}}^{\text{act}} + N_X \right) \cdot \frac{7}{8} \left( \frac{4}{11} \right)^{4/3} \right]
\]

\[
N_{\text{eff}}
\]
A few remarks on $N_{\text{eff}}$

- is not a constant, in general
  - increase through light decay products of massive particle
  - decrease when particles go non-relativistic
  - (in fact, technically $N_{\text{eff}} \leq 1$ today)

$N_{\text{eff}}$ can be < 3.046 at early times if neutrinos out of equilibrium; e.g., low reheating temperature:

[Ichikawa, Kawasaki, Takahashi (2005)]
Determining $N_{\text{eff}}$ from observation

Decoupling (T~1 eV)
- Cosmic Microwave Background anisotropies
- Large scale structure

Big Bang Nucleosynthesis (T~1 MeV)
- Primordial element abundances
Decoupling
$N_{\text{eff}}$ and the CMB

CMB map

expand in spherical harmonics

CMB angular power spectrum

$\ell(\ell+1)C_\ell/(2\pi)$ vs. Multipole Moment ($\ell$)

[WMAP (2010)]
Angular power spectrum is a function of $O(10)$ cosmological parameters (e.g., $\omega_b$, $\omega_{dm}$, $\omega_\nu$, $\Omega_{de}$, $N_{\text{eff}}$, ...).
and the CMB

$N_{\text{eff}}$ and the CMB

- Physical impact of changing $N_{\text{eff}}$
  - Matter-radiation equality
  - Sound horizon
  - Anisotropic stress
  - Damping tail

- All of these effects can to some extent be mimicked by adjusting other parameters

[Seljak & Bashinsky (2003), Hou et al. (2010)]
Matter-radiation equality

\[ 1 + z_{eq} = \frac{\Omega_m}{\Omega_r} \approx \frac{\Omega_m h^2}{\Omega_\gamma h^2} \frac{1}{1 + 0.2271 N_{\text{eff}}} \]

Larger $N_{\text{eff}}$ → later equality → enhanced early integrated Sachs-Wolfe-effect → first/higher peak ratio larger
Matter-radiation equality

\[ 1 + z_{\text{eq}} = \frac{\Omega_m}{\Omega_r} \sim \frac{\Omega_m h^2}{\Omega_\gamma h^2} \frac{1}{1 + 0.2271 N_{\text{eff}}} \]

- Larger \( N_{\text{eff}} \) \( \rightarrow \) later equality \( \rightarrow \) enhanced early integrated Sachs-Wolfe-effect \( \rightarrow \) first/higher peak ratio larger

adjust \( \omega_m \) \( \rightarrow \) keep \( z_{\text{eq}} \) constant
Sound horizon

\( \theta_s = \text{Sound horizon/distance to last scattering surface} \)
→ determines positions of acoustic peaks

\[
\theta_s \propto \frac{\Omega_m^{-1/2}}{\int_{a_s}^1 \frac{da}{a^2 \sqrt{\Omega_m a^{-3} + (1 - \Omega_m)}}}
\]

\[
\Omega_m = \frac{\omega_m}{h^2}
\]  
[Abazajian et al. (2012)]
Sound horizon

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\[ \theta_s \propto \frac{\Omega_m^{-1/2}}{\int_{a_*}^1 \frac{da}{a^2 \sqrt{\Omega_m a^{-3} + (1 - \Omega_m)}}} \]

\[ \Omega_m = \frac{\omega_m}{h^2} \]

[Abazajian et al. (2012)]

- adjust Hubble parameter
- keep \( \theta_s \) constant
Anisotropic stress

- Neutrinos are decoupled \(\rightarrow\) free streaming \(\rightarrow\) anisotropic stress
- Dampens fluctuations during radiation domination
- Suppression of power at multipoles > 200
Anisotropic stress

- Free streaming particles $\rightarrow$ anisotropic stress
- Dampens fluctuations during radiation domination
- Suppression of power at multipoles $> 200$

$A_s$, $n_s$ can have similar effect
Anisotropic stress

- Free streaming particles $\rightarrow$ anisotropic stress
- Dampens fluctuations during radiation domination
- Suppression of power at multipoles $> 200$

Adjust $A_s$, $n_s$ → primordial fluctuations can have similar effect
Damping tail

- Last scattering surface has finite thickness
  \[\rightarrow\] exponential damping of fluctuations below damping scale
$N_{\text{eff}}$ from WMAP+LSS+...

- lower limit from WMAP alone (→ anisotropic stress)
- meaningful upper limit requires combination with other data sets sensitive to matter density and expansion rate ...

[WMAP: Komatsu et al. (2008)]
$N_{\text{eff}}$ from CMB alone

... or measurement of the damping tail of the CMB angular power spectrum

[Keisler et al. (2011)]

[WMAP]

[WMAP+ACT]

[WMAP+ACT+BAO+H]

[South Pole Telescope]

[Atacama Cosmology Telescope]
CMB+LSS constraints

- Depending on data sets, $N_{\text{eff}} > 3.046 \, @ \, \sim 1.5 - 2.5\sigma$
- Result appears to be robust under different statistical methods, more complex cosmological models
- Bias due to potentially insufficient treatment of CMB secondaries (unresolved point sources, radio sources, infrared background, kSZ, tSZ) or beam uncertainties remains a possibility

[JH (2011), Calabrese et al. (2011)]
The next milestone ...
Launched 9th May 2009
Data taking (almost) finished, analysis in progress
→ cosmology papers in January 2013
9 frequency channels 30-857 GHz
~ 5 arcmin resolution
→ limited by cosmic variance up to multipoles of ~2000
Expected sensitivity to $N_{\text{eff}}$: $\sigma_{N_{\text{eff}}} \approx 0.2-0.3$

[JH, Lesgourgues, Mangano (2007)]
Big Bang Nucleosynthesis
BBN

Boltzmann equation

nuclear interaction rates ↔ expansion rate

$T$ ~ 1 MeV

$T$ ~ 0.2 MeV

$n + \nu_e \rightleftharpoons p + e^-$

neutrons and protons in equilibrium

$n \rightarrow p + e^- + \bar{\nu}_e$

neutrons start decaying

$\rightarrow D, {}^4\text{He}, \ldots$

most surviving neutrons end up in $^4\text{He}$
BBN

Boltzmann equation

nuclear interaction rate $\Gamma(\omega_b, f_{\nu_e})$ \iff expansion rate $H \propto \sqrt{\rho_r}$

primordial element abundances as function of $(\omega_b, f_{\nu_e}, N_{\text{eff}}, \ldots)$
BBN

- Assume standard BBN with standard active neutrino sector $\rightarrow N_{\text{eff}} > 3.406$
  (would have to make assumptions about momentum distribution of electron neutrinos otherwise, since they participate in nuclear reactions)
- Define $N_s \equiv N_{\text{eff}} - 3.046$
Measure primordial abundances → infer $N_s$
BBN

E.g., Helium:
- increasing radiation density
  → higher expansion rate
  → n-p freeze-out at higher T
  → n/p = exp[-Δm/T]
  → greater Helium abundance

Measure primordial abundances → infer $N_s$
BBN constraints: Deuterium + Helium

\[ N_S < 1.26 \ (1.24) \ @95\% \ credibility \]

\[ \text{Best-fit at } N_S = 0.86 \]

including CMB+LSS prior on baryon density

[JH, Hannestad, Raffelt, Wong (2011)]
BBN constraints

- One additional species ok, even slightly preferred
- Two or more excluded at high significance

[JH, Hannestad, Raffelt, Wong (2011)]
III. Sterile neutrino hot dark matter?
Hints for sterile neutrinos?

Observations at odds with standard 3-neutrino interpretation of global oscillation data

- LSND anomaly [Aguilar (2001)]
- MiniBooNE antineutrino results [Aguilar-Arevalo (2010)]
- Short-baseline reactor experiments (Bugey, ROVNO, Krasnoyarsk, ILL, Gösgen)
  - Recent re-evaluation of reactor fluxes [Mention et al. (2011)]
Hints for sterile neutrinos?

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Can possibly resolved with oscillations into sterile neutrinos with $\Delta m^2 \sim eV^2$
Hints for sterile neutrinos?

- Results from reactor/global fit:

Table I: Best fit points for the 3+1 and 3+2 scenarios from reactor anti-neutrino data.

|       | $\Delta m^2_{31} [eV^2]$ | $|U_{e3}|$ | $\Delta m^2_{21} [eV^2]$ | $|U_{e2}|$ | $\chi^2$/dof |
|-------|--------------------------|-----------|--------------------------|-----------|--------------|
| 3+1   | 1.78                     | 0.151     | 50.1                     | 67        |
| 3+2   | 0.46                     | 0.108     | 0.89                     | 0.124     | 46.5/65      |

Table II: Parameter values and $\chi^2$ at the global best fit points for 3+2 and 1+3+1 oscillations ($\Delta m^2$'s in eV$^2$).

|       | $\Delta m^2_{21}$ | $|U_{e3}|$ | $\Delta m^2_{31}$ | $|U_{e2}|$ | $\delta/\pi$ | $\chi^2$/dof |
|-------|-------------------|-----------|-------------------|-----------|-------------|--------------|
| 3+2   | 0.47              | 0.128     | 0.165             | 0.87      | 0.138       | 0.148        | 1.64         | 110.1/130    |
| 1+3+1 | 0.47              | 0.129     | 0.154             | 0.87      | 0.142       | 0.163        | 0.35         | 106.1/130    |

[Kopp, Maltoni, Schwetz (2011)]
Sterile neutrino scenario

Two qualitatively different mass hierarchies:

- "3+N"
- "N+3"

[JH, Hannestad, Raffelt, Tamborra, Wong (2010)]
Sterile neutrino scenario

- 3+1, 3+2, 3+3 are fine ... 
  ... as long as the steriles are light enough!

- Unfortunately, 1 eV appears to be somewhat too heavy

[JH, Hannestad, Raffelt, Tamborra, Wong (2010)]
Sterile neutrino scenario

Slightly problematic:
- BBN: too many
- CMB: too heavy

→ assuming standard cosmology and fully thermalised steriles

Solutions?
- Change cosmological model to weaken constraints, e.g.,
  - Degenerate BBN
  - LCDM + w + N_{massless}


- Have a closer look at production of steriles
Sterile neutrino production

- Start with active neutrinos in thermal equilibrium, no steriles
- Sterile states will be populated through oscillations
- Solve quantum kinetic equations until neutrinos decouple

[Hannestad, Tamborra, Tram (2012)]
Sterile neutrino production

For 1+1 model:
Complete thermalisation if mixing/mass splitting as required by oscillation experiments

\[ \delta N_{\text{eff}} \]

[Hannestad, Tamborra, Tram (2012)]
Sterile neutrino production

- Presence of large-ish initial lepton asymmetry could suppress production of steriles (e.g., for L=0.01):

$$\delta N_{\text{eff}}$$

[Hannestad, Tamborra, Tram (2012)]
Conclusions

- Cosmological observations are a powerful tool to constrain properties of the dark sector, such as neutrino masses or the velocity dispersion of dark matter.
- Some indication for the presence of an additional, relativistic dark component.
- Light sterile neutrinos are a possible candidate, but not entirely unproblematic.
- Exciting results from PLANCK soon!