QCD Matrix Elements And Truncated Showers

Or how to simulate multijet events at hadron colliders

Steffen Schumann

Institut für Theoretische Physik

Universität Heidelberg

Overview

- Motivation: Why consider multijet final states?
- The challenge of modelling them
- Combining tree-level matrix elements and parton showers
  - A new algorithm[a]
  - Systematics
- Validation and Application
  - Drell-Yan process at Tevatron
  - WBF Z-boson production at LHC
- Summary and Outlook

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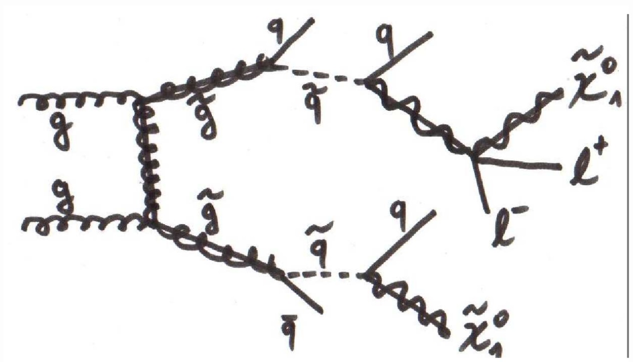
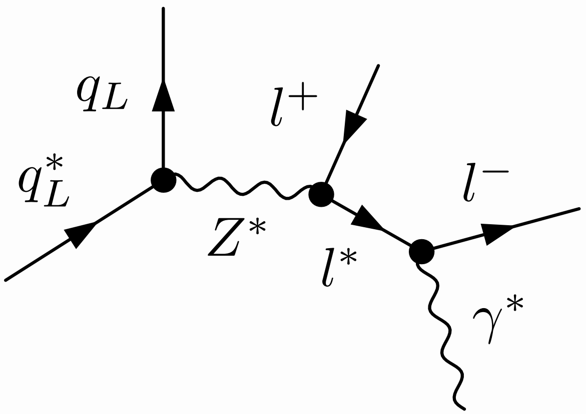
[a] Höche, Krauss, S., Siegert arXiv:0903.1219 [hep-ph]

Multijet Final States At Hadron Colliders

How is the electroweak symmetry broken?
Hierarchy between $v = 246\ \mathrm{GeV}$ and $M_{\mathrm{Planck}} = 10^{19}\ \mathrm{GeV}$?
Nature of Dark Matter?

$\rightsquigarrow$ **probe TeV scale physics: Higgs boson(s), SUSY, ED, little Higgs, ...**





- signals: leptons + n-jets + $\not{E}_T$
- backgrounds: V+jets, VV+jets, $t\bar{t}$+jets, QCD jets
- jet properties depend on nature of new physics [energies, flavours, kinematics]

$\rightsquigarrow$ **wide range of signatures – sophisticated simulation tools needed**

Multijet Final States At Hadron Colliders

Maybe nature doesn't share our prejudices?

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- $\rightsquigarrow$ **We have powerful machines though!**
- $\rightsquigarrow$ **Sizeable rates for producing new colour charged states!**

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Strawman: Scaled-up QCD [Kilic, Okui, Sundrum '08; Kilic, S., Son '08]

- extend SM by set of fermions charged under QCD and HyperColour

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4}H_{\mu\nu}H^{\mu\nu}$$

$\rightsquigarrow$ hypercolour confining at scale Λ_{HC}

- below Λ_{HC} bound states $\bar{\psi}\psi$ exist charged under QCD, e.g.
  - $SU(3)_c$ adjoint vector $\tilde{\rho}_\mu$ (coloron)
  - $SU(3)_c$ adjoint scalar $\tilde{\pi}$ (hyperpion)
- quantitative: $\psi'\mathrm{s} = (3, \bar{3})$ charged under $SU(3)_{QCD} \otimes SU(3)_{HC}$ only

$\rightsquigarrow$ LHC signatures: $pp \to \tilde{\pi}\tilde{\pi}$ & $pp \to \tilde{\rho}\tilde{\rho}$ with $\tilde{\pi} \to gg$ & $\tilde{\rho} \to \tilde{\pi}\tilde{\pi}$

$\rightsquigarrow$ resonance search in four- & eight-jet final states

Multijet Final States At Hadron Colliders

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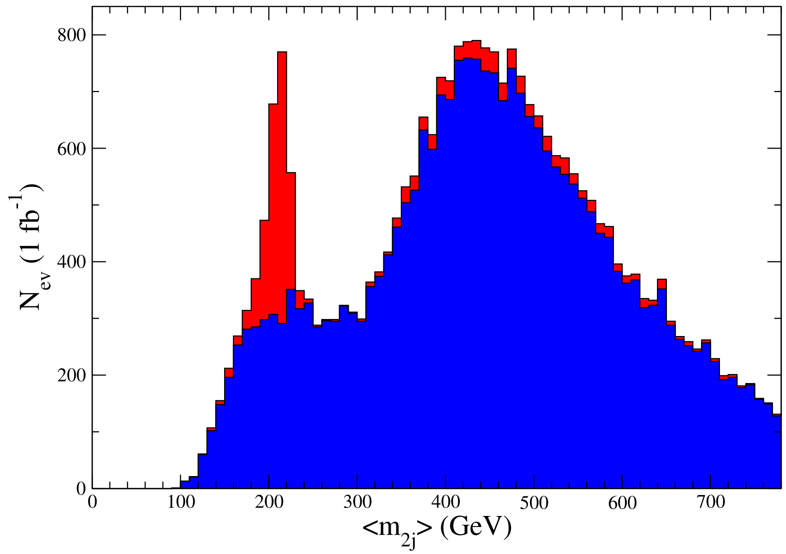
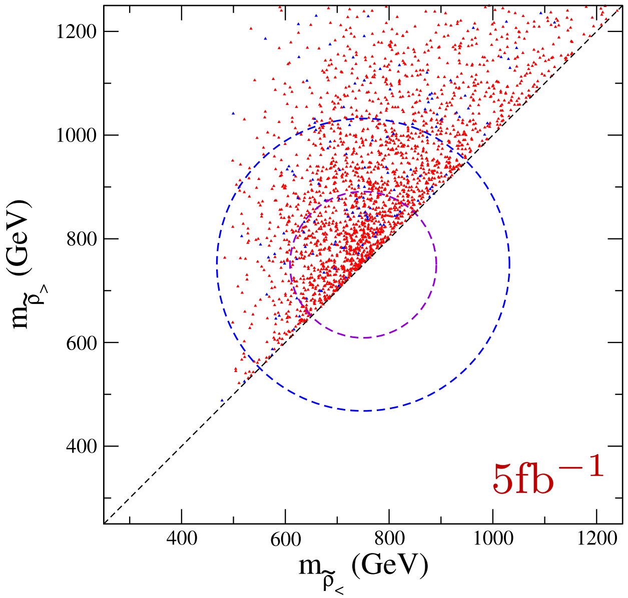
Strawman: Scaled-up QCD [Kilic, Okui, Sundrum '08; Kilic, S., Son '08]

benchmark model: $m_{\tilde{\pi}} = 225\ \mathrm{GeV}$ and $m_{\tilde{\rho}} = 750\ \mathrm{GeV}$

$\sigma_S = 2.2\ \mathrm{pb},\ \sigma_{\mathrm{BG}} = 24\ \mathrm{pb}$

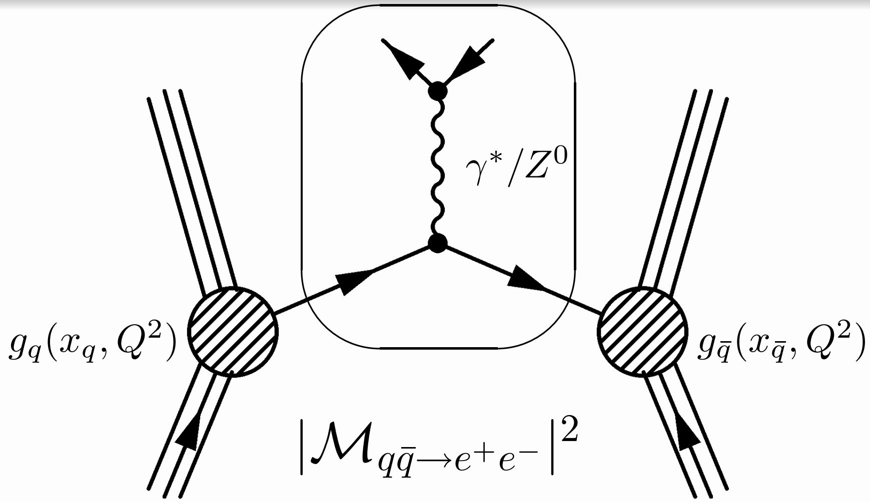


$\sigma_S = 470\ \mathrm{fb},\ \sigma_{\mathrm{BG}} = 29\ \mathrm{fb}$



➔ interesting to look into jets-only final states [tools are prepared]

Hadronic Cross Section: Drell-Yan Production



$$\sigma_{pp\to e^+e^-}(Q^2) = \sum_q \int \mathrm{d}x_q \mathrm{d}x_{\bar{q}}\; g_q(x_q, Q^2) g_{\bar{q}}(x_{\bar{q}}, Q^2)\; \mathrm{d}\hat{\sigma}_{q\bar{q}\to e^+e^-}$$

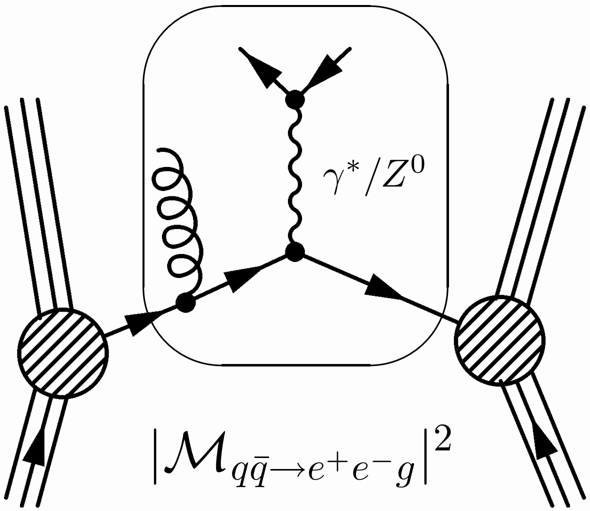
- partonic matrix element

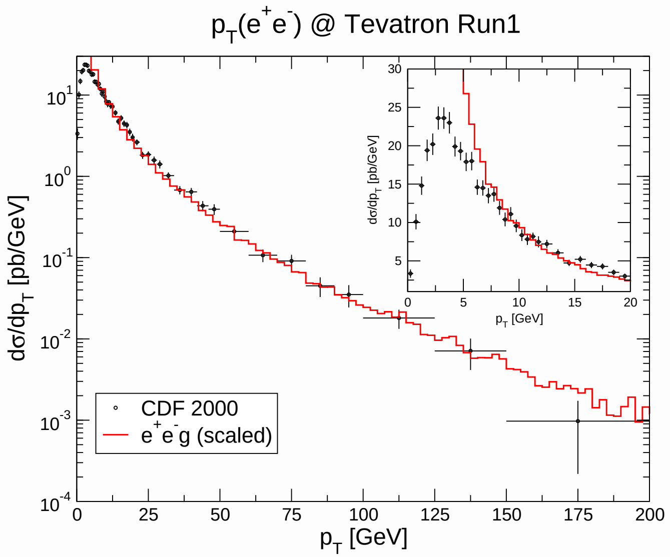
$$\mathrm{d}\hat{\sigma}_{q\bar{q}\to e^+e^-} = |\mathcal{M}_{q\bar{q}\to e^+e^-}|^2\; \mathrm{d}LIPS$$

- universal PDF $g_{q/g}(x, Q^2)$
- resummation of soft/collinear initial-state radiation in PDFs

$\rightsquigarrow$ **factorization into hard and soft contribution at Q^2**

Hadronic Cross Section: Drell-Yan Production





- cross section enhanced for small gluon p_T [plus soft emissions]

$$|\mathcal{M}_{q\bar{q}\to e^+e^-g}|^2 \sim |\mathcal{M}_{q\bar{q}\to e^+e^-}|^2 \frac{\alpha_S(\mu_R^2)}{p_T^2} \rightsquigarrow \sigma_{pp\to e^+e^-g} \sim \sigma_{pp\to e^+e^-}\alpha_S(\mu_R^2)\,\log\frac{p_T^{\max}}{p_T^{\min}}$$

$\rightsquigarrow |\mathcal{M}|^2$ factorize in IR limes (universal) $\rightsquigarrow$ large logs need to be resummed to all orders

punchline

- proper description of soft/collinear *and* hard emissions
- combine QCD matrix elements of different parton multiplicity with showers

[CKKW: Catani et. al ’01, MLM: Mangano et. al ’01, CKKW-L: Lönnblad ’01]

Formalism: Parton Showers

QCD evolution of PDFs

$$\frac{\partial}{\partial \log(t/\mu^2)} \frac{g_a(z,t)}{\Delta_a(\mu^2,t)} = \frac{1}{\Delta_a(\mu^2,t)} \int_z^{\zeta_{\max}} \frac{\mathrm{d}\zeta}{\zeta} \sum_{b=q,g} \mathcal{K}_{ba}(\zeta,t)\, g_b(z/\zeta,t)$$

$$\Delta_a(\mu^2,t) \;=\; \exp\left\{ -\int_{\mu^2}^{t} \frac{\mathrm{d}\bar{t}}{\bar{t}} \int_{\zeta_{\min}}^{\zeta_{\max}} \mathrm{d}\zeta \sum_{b=q,g} \frac{1}{2}\, \mathcal{K}_{ba}(\zeta,\bar{t}) \right\}$$

- Kernel describes parton splitting: $\mathcal{K}_{ba}(z,t) \stackrel{\mathrm{IR}}{\rightarrow} \frac{1}{\sigma_a^{(N)}(\Phi_N)} \frac{\mathrm{d}\sigma_b^{(N+1)}(z,t;\Phi_N)}{\mathrm{d}\log(t/\mu^2)\,\mathrm{d}z}$

  [IR (shower) factorization scheme]

- defines shower no-branch probabilities between two scales

$$\mathcal{P}^{(B)}_{\mathrm{no},\,a}(z,t,t') = \frac{\Delta_a(\mu^2,t')\, g_a(z,t)}{\Delta_a(\mu^2,t)\, g_a(z,t')} \quad \text{and} \quad \mathcal{P}^{(F)}_{\mathrm{no},\,a}(t,t') = \frac{\Delta_a(\mu^2,t')}{\Delta_a(\mu^2,t)}$$

$\rightsquigarrow$ backward evolution $\qquad\qquad$ $\rightsquigarrow$ forward evolution

Formalism: Matrix Elements And Parton Showers

construction criteria

- describe hardest emissions through full matrix elements

$$\mathcal{K}_{ba}(z,t) \to \frac{1}{\sigma_a^{(N)}(\Phi_N)}\,\frac{\mathrm{d}\sigma_b^{(N+1)}(z,t;\Phi_N)}{\mathrm{d}\log(t/\mu^2)\,\mathrm{d}z}$$

- preserve shower-evolution equation [logarithmic accuracy]

$\rightsquigarrow$ **slice emission phase space by parton-separation criterion $Q_{ba}(z,t)$**

Formalism: Matrix Elements And Parton Showers

Phase-space separation

$$\mathcal{K}_{ba}^{\rm PS}(z,t) = \mathcal{K}_{ba}(z,t)\,\Theta\left[Q_{\rm cut} - Q_{ba}(z,t)\right] \quad \leftarrow \text{shower regime}$$

$$\mathcal{K}_{ba}^{\rm ME}(z,t) = \mathcal{K}_{ba}(z,t)\,\Theta\left[Q_{ba}(z,t) - Q_{\rm cut}\right] \quad \leftarrow \text{matrix element regime}$$

$\Rightarrow Q_{ba}(z,t)$ has to identify logarithmically enhanced phase-space regions

Sudakov form factor and no-branch probabilities factorize

$$\Delta_a(\mu^2,t) = \Delta_a^{\rm PS}(\mu^2,t)\,\Delta_a^{\rm ME}(\mu^2,t)$$

$$\rightsquigarrow \mathcal{P}^{(B)}_{{\rm no},\,a}(t,t') = \mathcal{P}^{(B)\,{\rm PS}}_{{\rm no},\,a}(t,t')\,\mathcal{P}^{(B)\,{\rm ME}}_{{\rm no},\,a}(t,t') = \frac{\Delta_a^{\rm PS}(\mu^2,t')\,g_a(z,t)}{\Delta_a^{\rm PS}(\mu^2,t)\,g_a(z,t')}\,\frac{\Delta_a^{\rm ME}(\mu^2,t')}{\Delta_a^{\rm ME}(\mu^2,t)}$$

- need to veto shower emissions with $Q > Q_{\rm cut}$
- matrix elements need to be reweighted [made exclusive quantities]

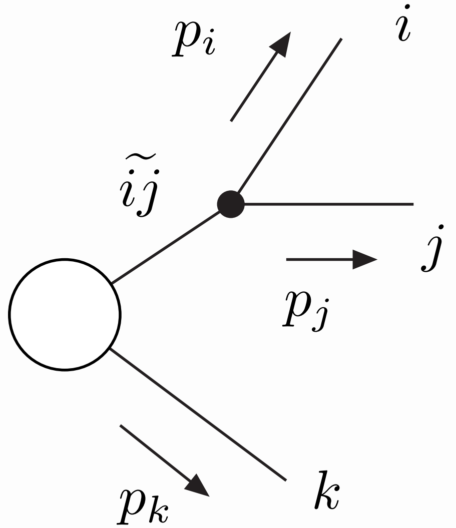
  $\hookrightarrow$ think of ME's as predetermined shower emissions, truncated shower

Formalism: The Phase-Space Separation Criterion (I)

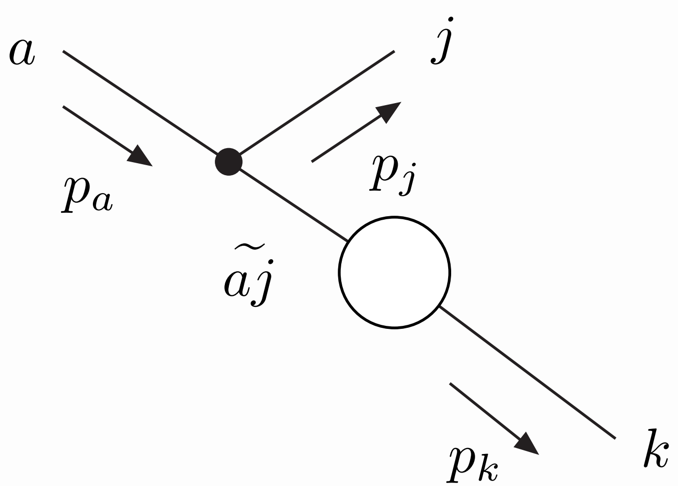
new proposal for phase-space separation [Catani–Seymour inspired]

$$Q^2_{ij} \;=\; 2\,p_i p_j \;\min_{k\neq i,j}\; \frac{2}{C^k_{i,j} + C^k_{j,i}}$$

$\hookrightarrow$ minimize over colour partners k



final-state partons $(ij) \to i,\ j$



initial-state parton $a \to (aj)\,j$

$$C^k_{i,j} \;=\; \begin{cases} \dfrac{p_i p_k}{(p_i+p_k)p_j} - \dfrac{m_i^2}{2\,p_i p_j} & \text{if } j = g \\[2ex] 1 & \text{else} \end{cases}$$

$$C^k_{a,j} \;=\; C^k_{(aj),\,j}$$

with $p_{aj} = p_a - p_j$

Formalism: The Phase-Space Separation Criterion (II)

soft limit: $p_j = \lambda q$, $\lambda \to 0$

$$\frac{1}{Q_{ij}^2} \to \frac{1}{2\,\lambda^2}\,\frac{1}{2\,p_i\,q}\,\max_{k\neq i,j}\left[\frac{p_i p_k}{(p_i+p_k)\,q} - \frac{m_i^2}{2\,p_i q}\right]$$

quasi-collinear limit: $k_\perp \to \lambda k_\perp$, $m \to \lambda m$

$$\frac{1}{Q_{ij}^2} \to \frac{1}{2\,\lambda^2}\,\frac{1}{p_{ij}^2 - m_i^2 - m_j^2}\left(\tilde{C}_{i,j} + \tilde{C}_{j,i}\right)$$

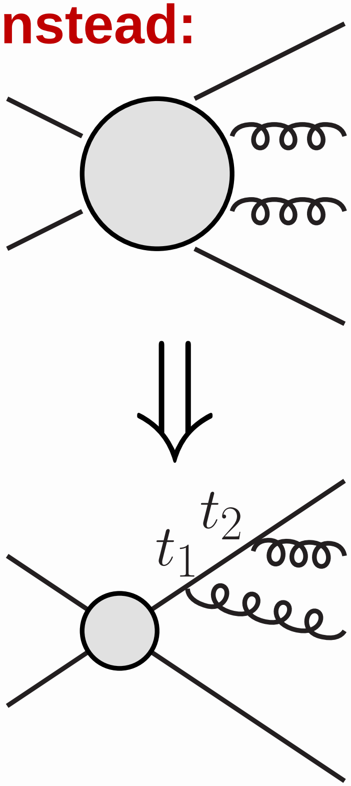
where

$$\tilde{C}_{i,j} = \begin{cases} \dfrac{z}{1-z} - \dfrac{m_i^2}{2\,p_i p_j} & \text{if } j = g \\ 1 & \text{else} \end{cases}$$

➔ measure correctly identifies enhanced phase-space regions

Monte Carlo Algorithm

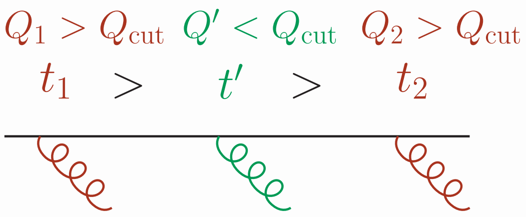
sadly we can't directly generate MEs in shower scheme, instead:



1. evaluate n-jet MEs, regularized by $Q_{\rm cut}$, at μ_F^2 & μ_R^2
2. generate ME configuration according to σ_n and $d\sigma_n$
3. for given ME configuration reconstruct most probable shower branching history
   - cluster backwards using shower kernels & inverted kinematics
   - chain of ME 'emissions' in terms of shower variables (t_i, z_i, ϕ)
4. reweight event with factors $\alpha_S(\mu_i^2)/\alpha_S(\mu_R^2)$, $\mu_i^2 = \mu^2(t_i, z_i)$
5. start shower for core process at $t_{\rm max}$, i.e. μ_F^2
6. *veto the event* if shower generates emission above $Q_{\rm cut}$

   [accounts for $\mathcal{P}_{\rm no}^{(B)\,{\rm ME}}$, equivalent to analytic Sudakov reweighting]
7. insert ME branchings when crossing corresponding t_i

   $\rightsquigarrow$ intermediate lines radiate: truncated shower



Implementation

algorithm implemented in the Sherpa generator [Gleisberg et. al ’08]

- use new matrix-element generator Comix [Gleisberg, Höche ’08]
  - Berends-Giele recursion
  - colour-ordered amplitudes [straightforward large-N_c assignment]
- Catani–Seymour subtraction based shower [Krauss, S. ’07]
  - emitter–spectator notion
  - invariant splitting variables
  - local momentum recoil
  - soft-colour coherence inherent [dipole factorization]

$\rightsquigarrow$ combination provides optimal analytic control

Systematics: Matrix Elements And Parton Showers

merging related

- value of phase-space separation cut, Q_{cut}
- maximum number of jets from hard ME's, N_{max}
- choice of internal separation measure

pQCD related

- scale uncertainties from ME's
- scale uncertainties from PS's
- PDF uncertainties [enter in ME and PS]

others

- choice of the LO process [DY vs. DY+2jets, influences μ_F]

Validation: Drell-Yan Production At Tevatron

- Stability of cross sections?
- Observables dependent on merging cut?
- Comparison to data!

calculational setup

- $p\bar{p} \to e^+e^- + X$ with $66\ \mathrm{GeV} \leq M_{e^+e^-} \leq 116\ \mathrm{GeV}$
- $\mu_F^2 = M_{e^+e^-}^2$
- $N_{\mathrm{max}} = 0 \ldots 6$ & $Q_{\mathrm{cut}} = 20/30/45\ \mathrm{GeV}$

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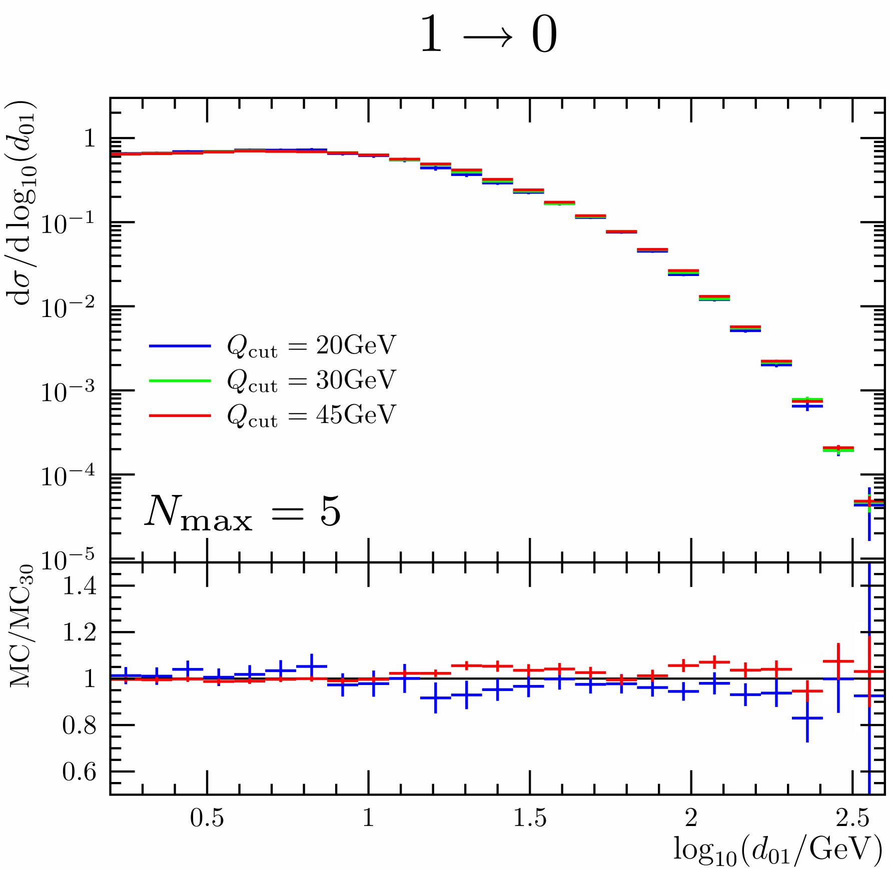
- $N_{\mathrm{max}} = 0 \ldots 6$ & $Q_{\mathrm{cut}} = 20/30/45\ \mathrm{GeV}$

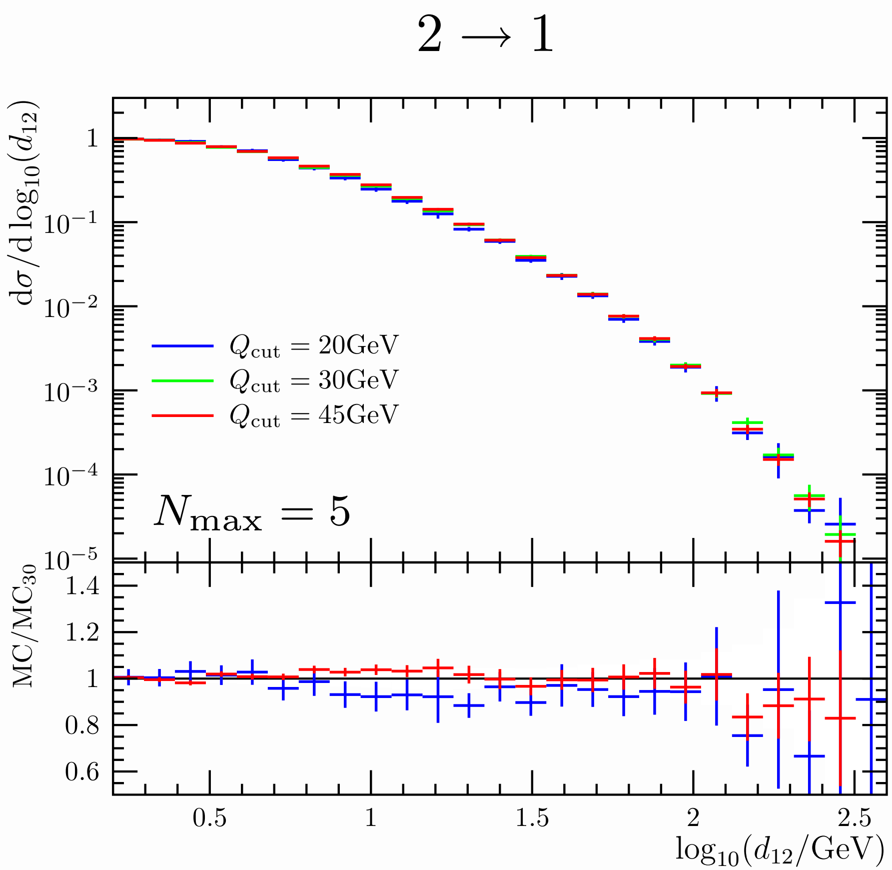
		N_{max}						
		0	1	2	3	4	5	6
Q_{cut}	20 GeV	192.6(1)	191.0(3)	190.5(4)	189.0(5)	189.4(7)	188.2(8)	189.9(10)
	30 GeV		192.3(2)	192.7(2)	192.6(3)	192.9(3)	192.7(3)	193.2(3)
	45 GeV		193.6(1)	194.4(1)	194.3(1)	194.4(1)	194.6(2)	194.4(1)

$\rightsquigarrow$ **"merging systematics" of $\sigma_{\mathrm{tot}} < \pm 3\%$**

Validation: Drell-Yan Production At Tevatron

- $Q_{\rm cut}$ variation: differential k_T jet rates

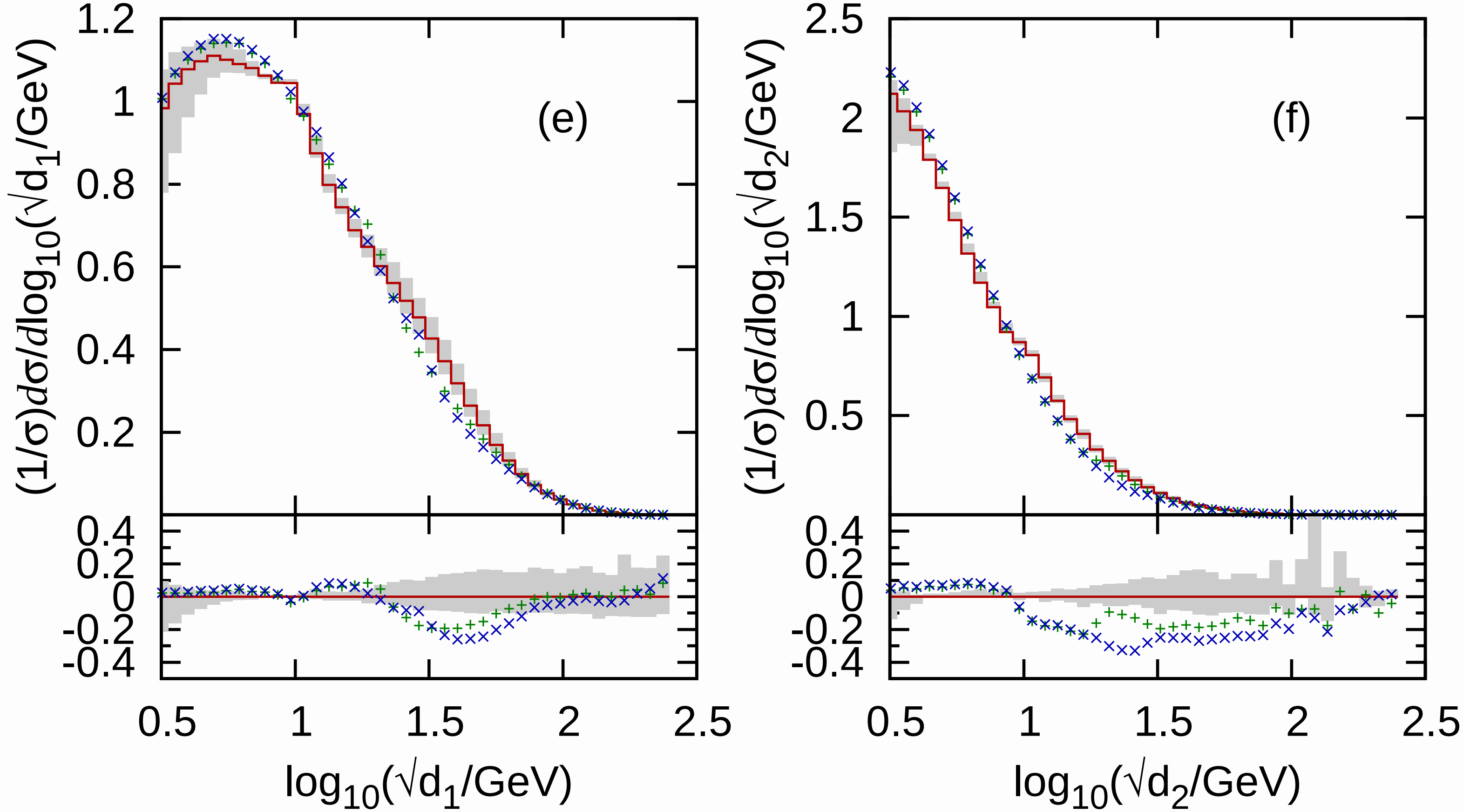




Validation: Drell-Yan Production At Tevatron

- Q_{cut} **variation: differential k_T jet rates**

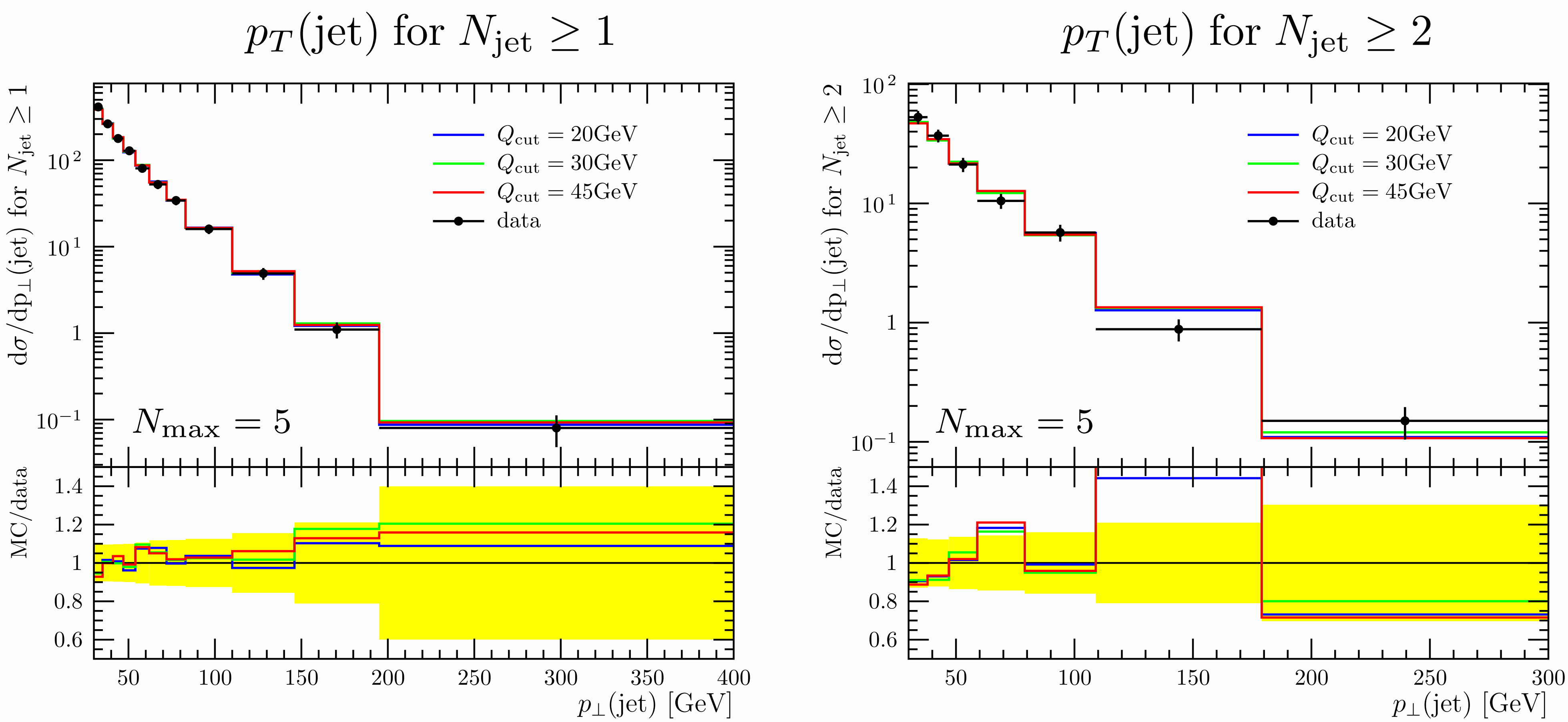
$\Rightarrow$ standard Sherpa CKKW for W+jets[a]



[a] taken from Alwall et. al Eur. Phys. J. C **53** (2008) 473

Validation: Drell-Yan Production At Tevatron

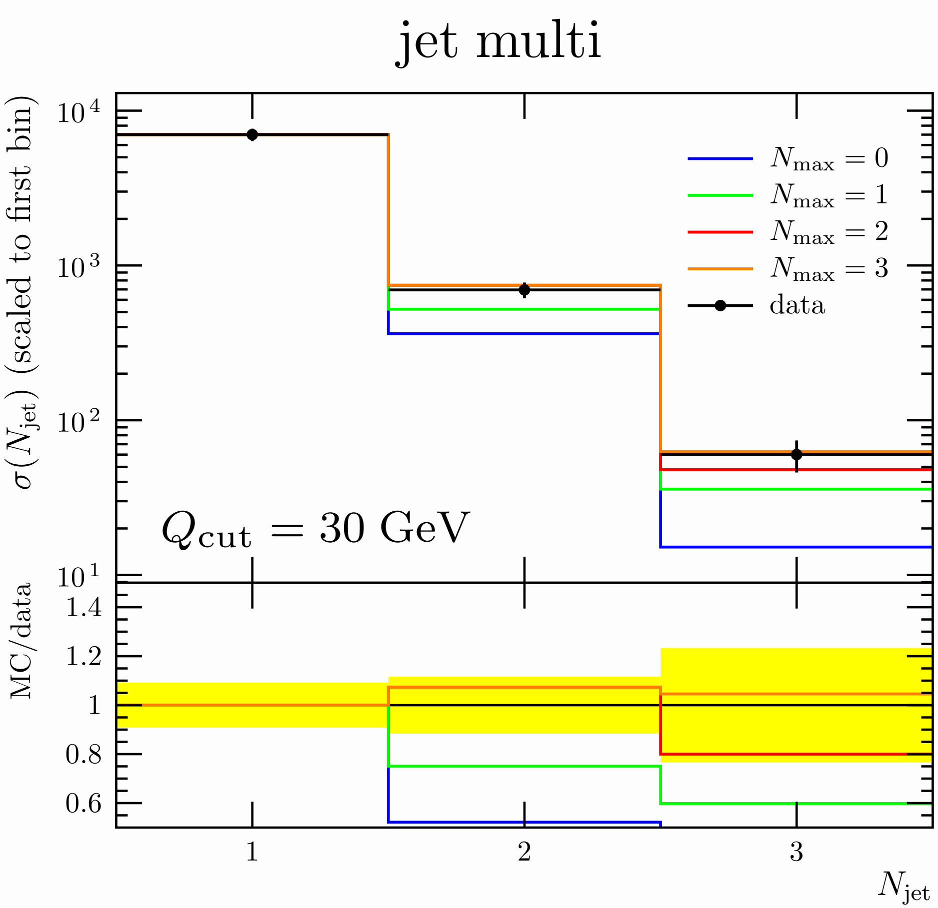
- Q_{cut} **variation: all-jet p_T spectra (data CDF '08)**

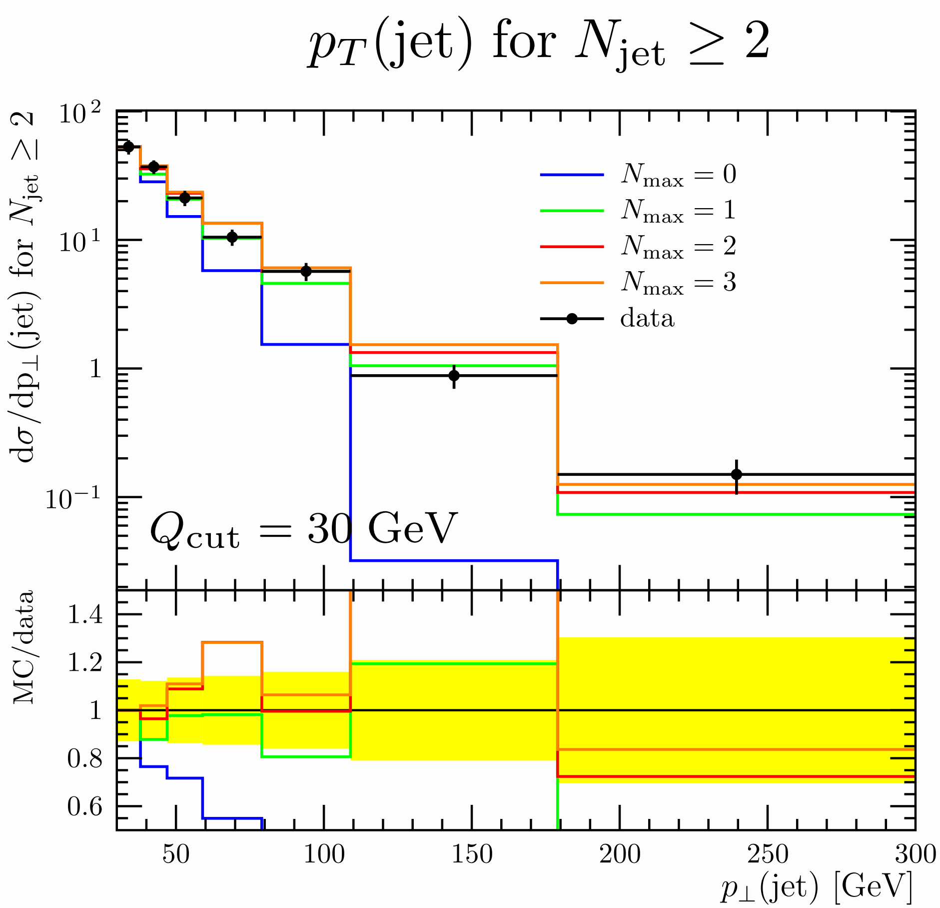


$\rightsquigarrow$ Q_{cut} **variations within $\pm 10\%$**

Validation: Drell-Yan Production At Tevatron

- N_{max} **variation: jet-multi & all-jet p_T (data CDF '08)**

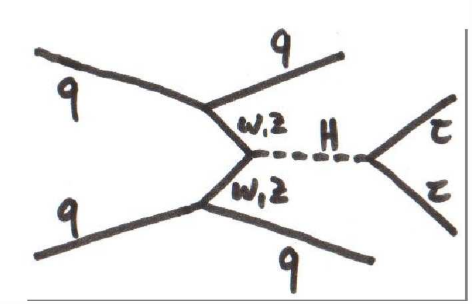


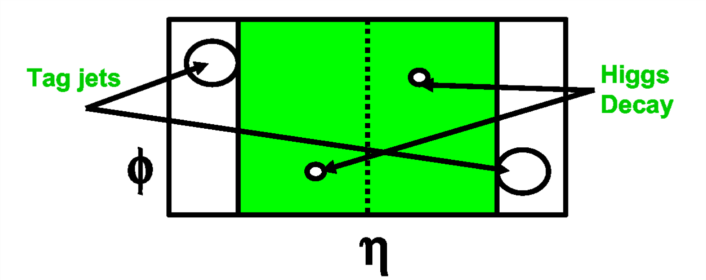


$\rightsquigarrow$ N_{max} **variation observable dependent**

Application: Weak-Boson-Fusion Process

Motivation: Higgs production in weak-boson-fusion [Rainwater,Zeppenfeld, ...]

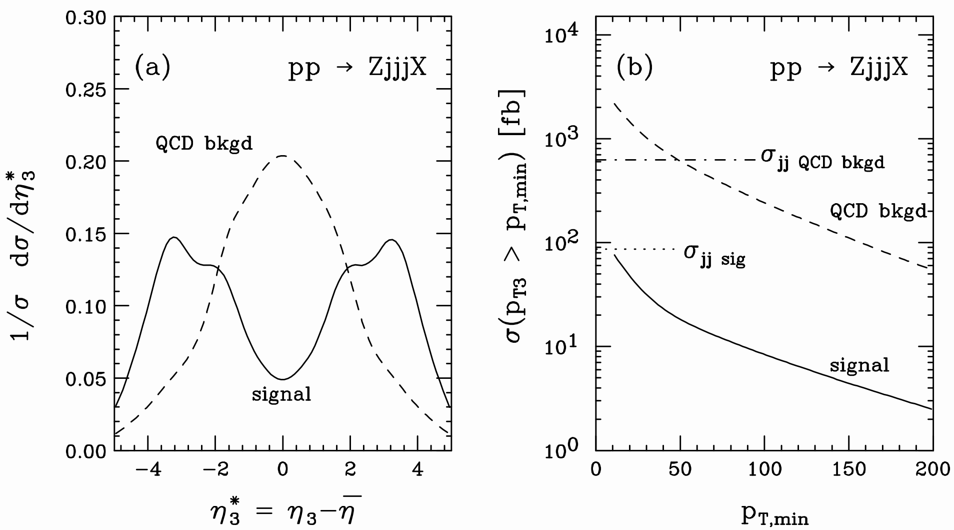




- rapidity gap between two forward/backward tagged jets
- signal/background ratio depends on central (mini) jet veto

Foreplay/Testbed: Z-boson production in WBF, $Z \to e^+e^-$

- Signal α_{EW}^4, BG $\alpha_{EW}^2\alpha_S^2$
- study central jet veto in ME+PS
  [Plehn, S., in preparation]
- compare to parton-level approx.
  [Rainwater, Szalapski, Zeppenfeld '96]



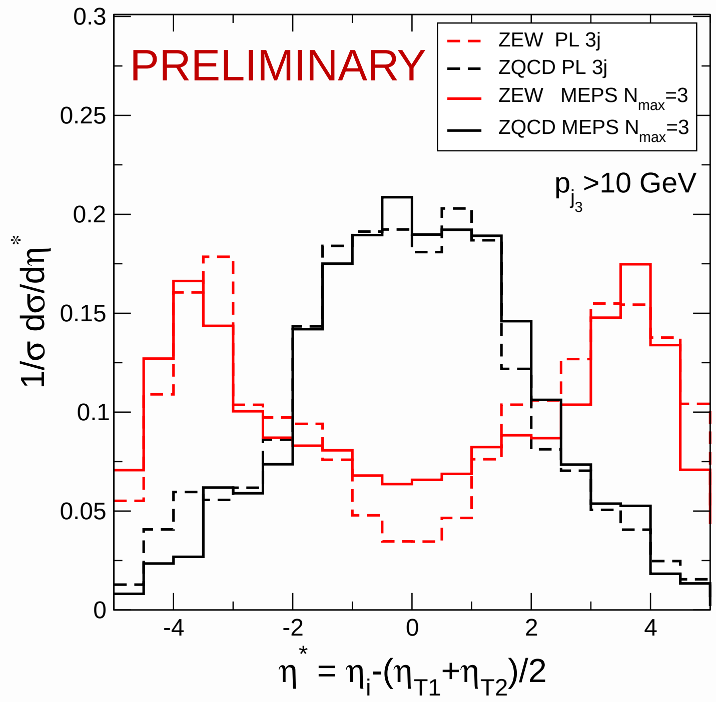
Application: Z-boson Production In WBF At LHC

minimal WBF cuts

$$p_{T\,j^{\rm tag}} \geq 40~\text{GeV}$$

$$\eta_{j_1^{\rm tag}} \cdot \eta_{j_2^{\rm tag}} \leq 0\,, \qquad \Delta\eta_{j_{1,2}^{\rm tag}} = |\eta_{j_1^{\rm tag}} - \eta_{j_2^{\rm tag}}| \geq 4.4$$

$$\eta_{j^{\rm tag},min} + 0.7 \leq \eta_{l_{1,2}} \leq \eta_{j^{\rm tag},max} - 0.7$$



jet activity in the central region

- PL estimate from exponentiation model

$$P_n = \frac{\bar{n}^n}{n!} e^{-\bar{n}} \quad \text{with} \quad \bar{n}(p_{T,\text{veto}}) = \frac{1}{\sigma_2} \int\limits_{p_{T,\text{veto}}}^{\infty} dp_{T,3}\, \frac{d\sigma_3}{dp_{T,3}}$$

$\hookrightarrow$ integrated over veto region

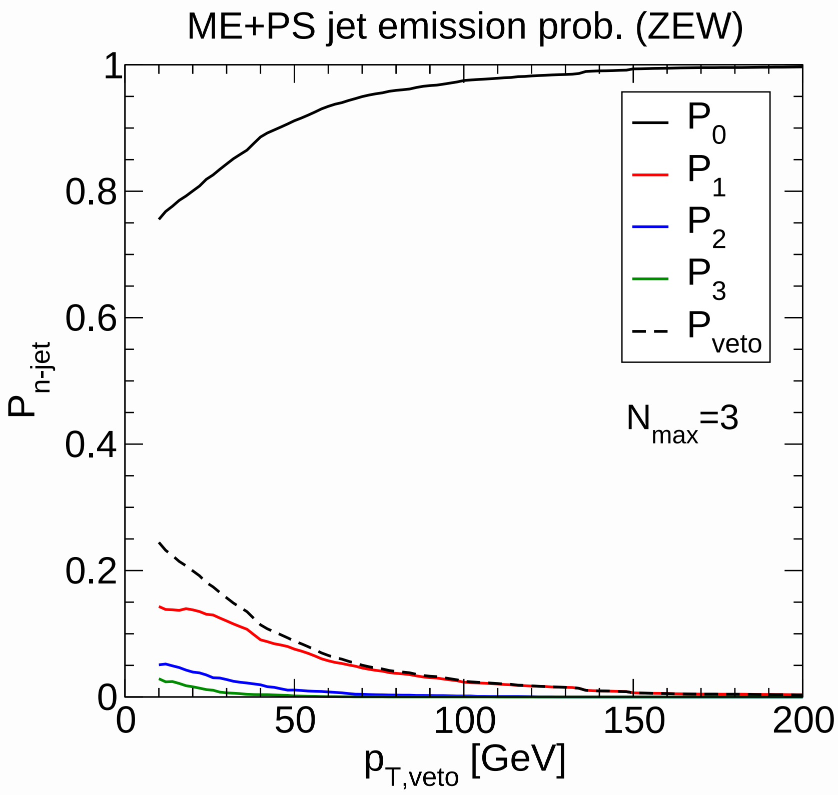
$$\rightsquigarrow P_{\text{veto}}(p_{T,\text{veto}}) = 1 - e^{-\bar{n}}$$

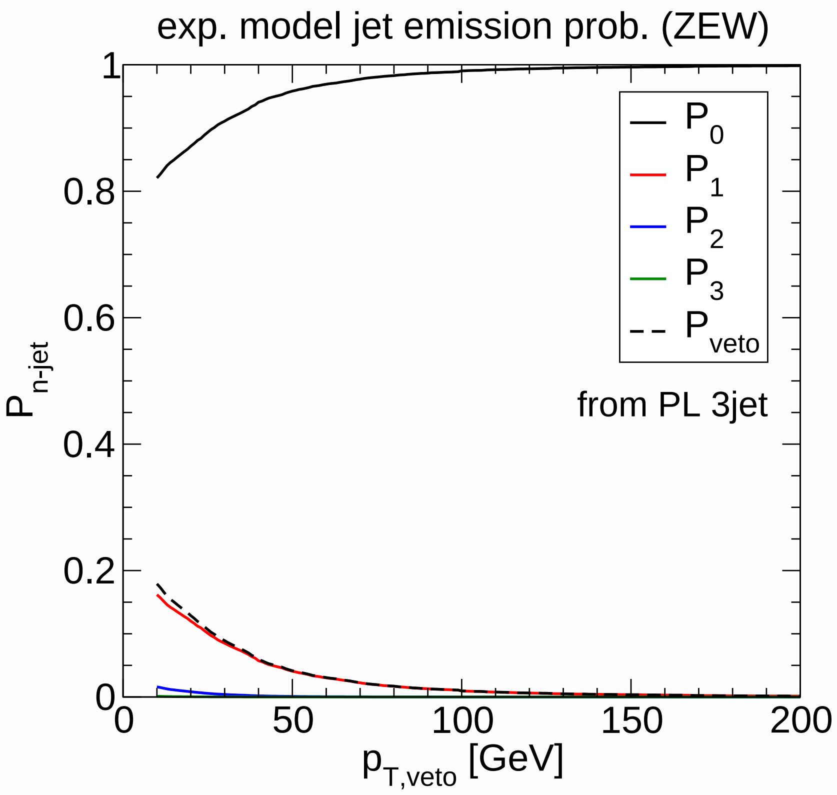
- compare to Monte Carlo predictions using merging approach

Application: Z-boson Production In WBF At LHC

- jet multiplicities for signal events as a function of $p_{T,\mathrm{veto}}$

PRELIMINARY



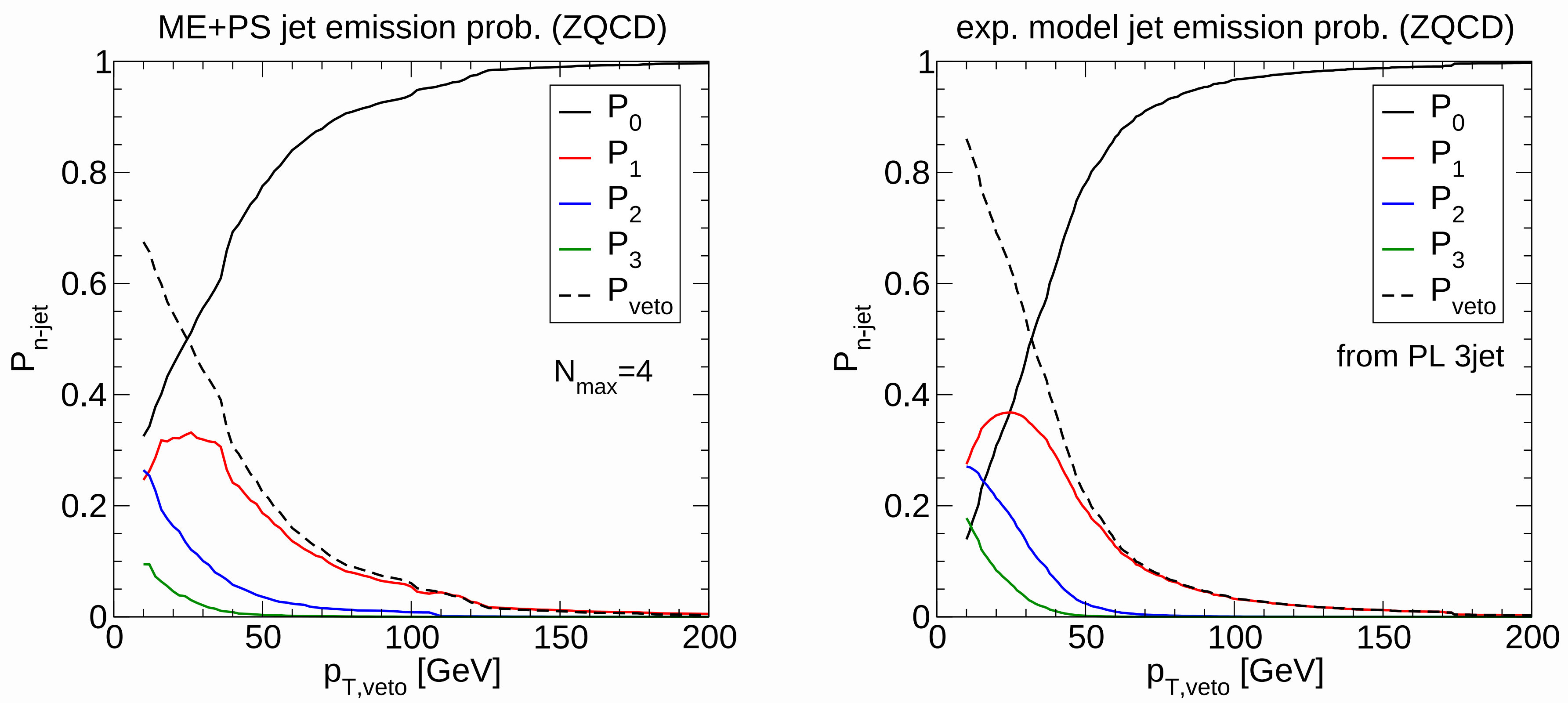


$\rightsquigarrow$ ME+PS predicts somewhat higher tails and more low-p_T multijets

Application: Z-boson production In WBF At LHC

- jet multiplicities for background events as a function of $p_{T,\text{veto}}$

PRELIMINARY



$\rightsquigarrow$ differences in veto probability at low-p_T due to $n_{\text{jet}}\big|_{\text{veto}} \geq 3$

Conclusions/Outlook

improved formalism for combining QCD matrix elements and showers

- proof of correctness in initial-state evolution
- better phase-space separation
- largely reduced merging systematics
- process and merging systematics can separately be assessed

⇝ calculational framework for multijet studies

ongoing/future directions

- mini-jet veto in WBF
- detailed systematics study for gauge-boson production
- release code with Sherpa-1.2
- extend formalism to include one-loop amplitudes