Exercise 1: Suppose the random variable $x$ is uniformly distributed in the interval $[\alpha, \beta]$, with $\alpha, \beta > 0$. Find the expectation value of $1/x$, and compare the answer to $1/E[x]$ using $\alpha = 1$, $\beta = 2$.

Exercise 2: Consider a random variable $x$ and constants $\alpha$ and $\beta$. Show that

$$
E[\alpha x + \beta] = \alpha E[x] + \beta, \\
V[\alpha x + \beta] = \alpha^2 V[x].
$$

(1)

Exercise 3: Consider two random variables $x$ and $y$.

(a) Show that the variance of $\alpha x + y$ is given by

$$
V[\alpha x + y] = \alpha^2 V[x] + V[y] + 2\alpha \text{cov}[x, y] \\
= \alpha^2 V[x] + V[y] + 2\alpha \rho \sigma_x \sigma_y,
$$

(2)

where $\alpha$ is any constant value, $\sigma_x^2 = V[x]$, $\sigma_y^2 = V[y]$, and the correlation coefficient is $\rho = \text{cov}[x, y]/\sigma_x \sigma_y$.

(b) Using the result of (a), show that the correlation coefficient always lies in the range $-1 \leq \rho \leq 1$. (Use the fact that the variance $V[\alpha x + y]$ is always greater than or equal to zero and consider the cases $\alpha = \pm \sigma_y/\sigma_x$.)

4: Suppose the independent random variables $x_1$ and $x_2$ have means $\mu_1 = \mu_2 = 10$ and variances $\sigma_1^2 = \sigma_2^2 = 1$. Use error propagation to find the variance of

$$
y = \frac{x_1}{x_2}.
$$

(3)

Comment on the validity of the procedure if one had $\mu_2 = 1$. 